Downwind

for

Dummies

How to launch a hot air balloon, kite downwind and catch that balloon again

Version 1.0

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How to catch a hot air balloon

This manual for catching hot air balloons will take You all the way from basic aerodynamic theories through steady state sailing or kiteboarding theory.
Aerodynamics as well as hydrodynamics of sailing and kiting are usually discussed using the wing theory for sail, kite keel or fins and even the edge of a kiteboard can be seen as a wing. The wing theory is a model theory. That means that it´s a simplified view upon the real thing. One example of a real effect that is not covered by wing theory is the Coriolis effect. Generally the wing theory could even be applied to supersonic sailing if there were such a thing. Then, however, it would have to be taken into account that there are massive changes in parameters like efficiencies, glide angles or-ratios and the like that are seen as constant parameters here most of the time. The applicability of a model theory must be checked and some limitations concerning applicability will be discussed.
The last chapter on the bauer cart and prop-missile deals with propeller driven craft. For propeller aerodynamics other model theories apply. However, the analogy between sailing craft and propeller driven craft will be scetched and feasibility of a propeller driven craft for going downwind faster than the wind itself will be discussed using wing theory most of the time for simplicity.
While chasing Your hot air balloon You might ask Yourself:
"How can i ride 40-50 mph in only 20-25 mph of wind?"
Please refer to the 4th chapter on kitesurfing theory.

Content:

Part 1: Kitesurfing basics

Speed triangles
Wind, Speed, Apparent wind, Heading and how they are related

Aerodynamic and hydrodynamic forces
Lift, drag, lift- and drag coefficients, glide angles and forces and how they are related to speed and apparent wind

The wind window
This coveres the kitesurfing or kiteflying specific aspects

Kitesurfing theory
How to reach the fastest possible boardspeed or the fastest downwind VMG and why

Part 2: Analogies

The threaded shaft
why a propeller like wing on a threaded shaft behaves very much like a kiteboard

Prop missile
The "Bauer cart", prop-missiles and how to understand a propeller driven vehicle going downwind faster than the speed of the wind

Part 3: Appendix Mathematics

Basic mathematical proofs
All there is needed to handle the mathematics that might be needed to understand this manual

How to draw a steady state performance circle
You could use it once You could draw it? There You go.

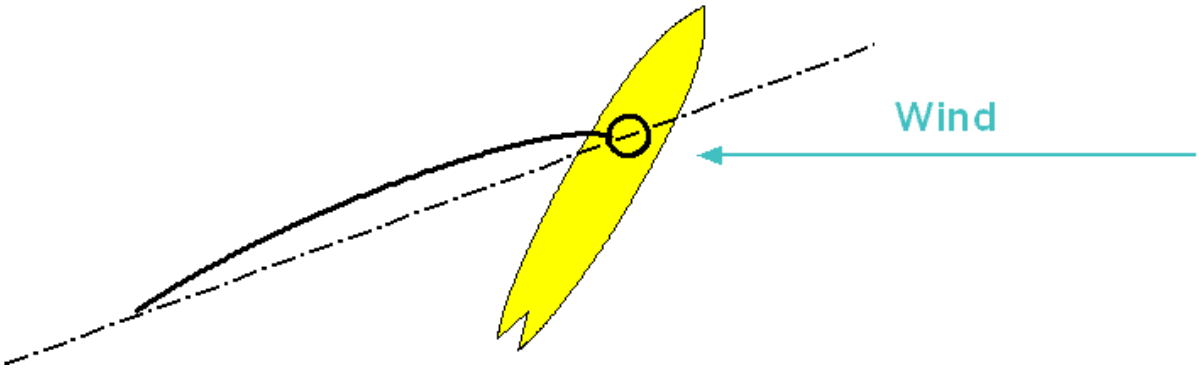
Points of special interest
Some special values for special courses and speeds

1 Kitesurfing basics

The kitesurfing basics deal with riding a kiteboard at a steady state. This is only a small part of kitesurfing since kiting gets much of it´s dynamics from flying the kite or steering the board dynamically. In speed kiting this dynamic is not to be seen. But not only speed kiting can be understood as an application of these basic kitesurfing physics but it will also show where the wind window will be during dynamic manouvers and why.

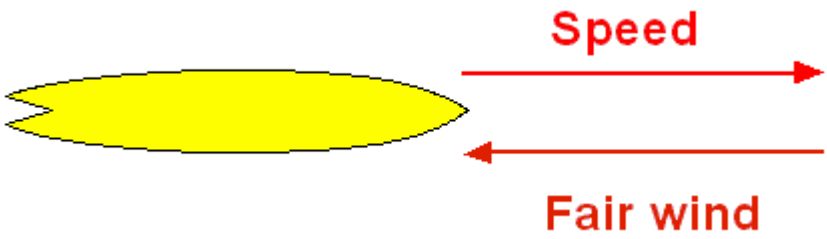
1.1 Speed triangles

A Sail is hit by the wind at standstill. With no motion of the ship there is nothing but the wind to be taken into account.



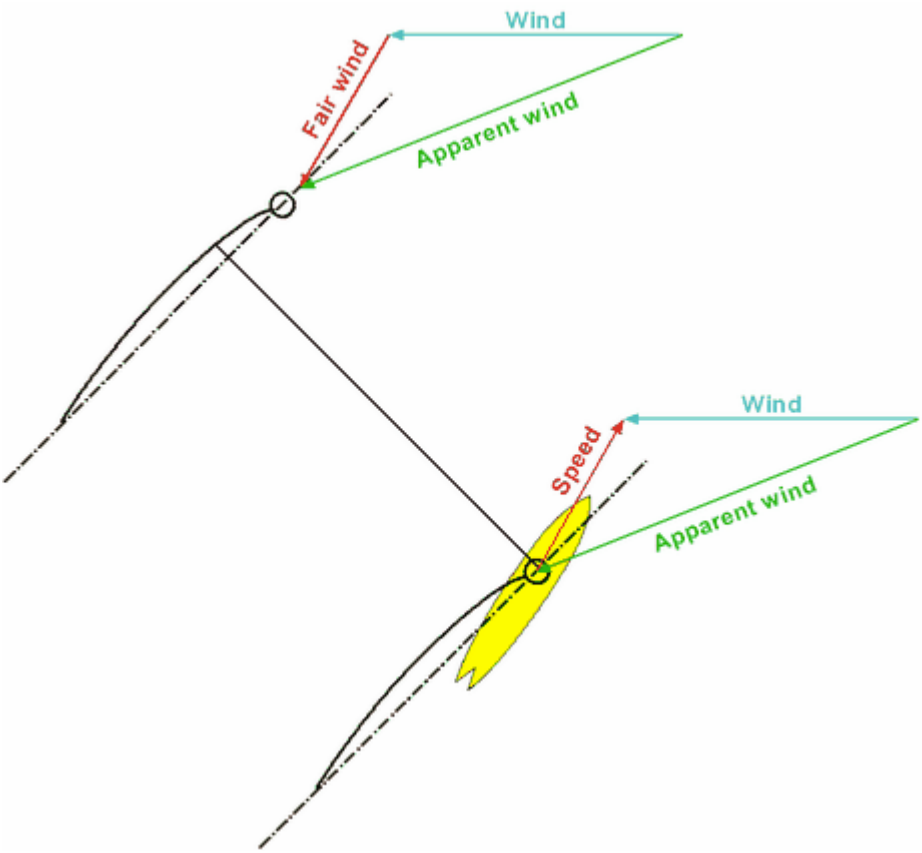
Wind with a motionless boat

This is not often the case for You might want to sail the ship with some speed. If You move to one direction You will be hit by the corresponding fair wind. The fair wind is antiparallel to the speed of the vehicle. The speed of the fair wind is exactly (per definition of the term) the same as the speed of Your motion. The direction of the fair wind is opposite to the direction of Your motion.



Fair wind

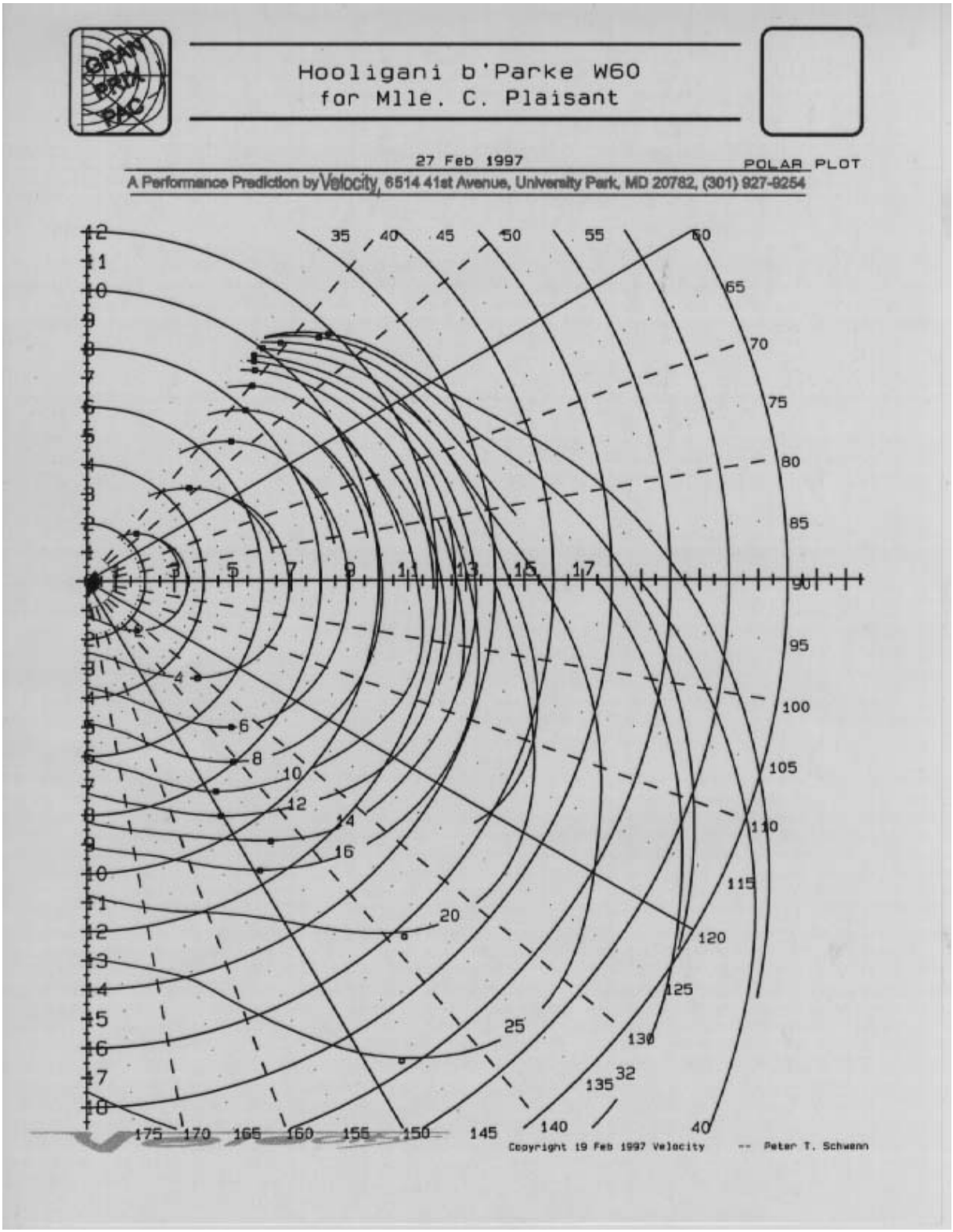
Now when moving a vehicle when there is wind both, the wind and the fair wind are to be taken into account. The airflow around the vehicle as it is felt by the crew and has to be taken as the airflow interfering with the sails or kite is called *apparent wind*.
As the vectoriel sum of two speeds principally leads to a triangle this dependancy is called the wind- or speed triangle.
As there are "two kinds of wind", the wind felt by a motionless spectator and the *apparent wind* felt if You move, the wind felt by a motionless sepectator is frequently adressed as *true wind* in sailing theory as well.



Speed triangle

The picture above shows a vehicle carrying a sail and a kite flown at steady state. Both move at the same speed so the apparent wind is the same. For the sail it is shown that the apparent wind is the vectorial subtraction of the speed from the wind.
For the kite it is demonstrated here that the apparent wind is the vectorial sum of wind and fair wind. Both ways of expressing how the apparent wind depends on wind and speed are common.

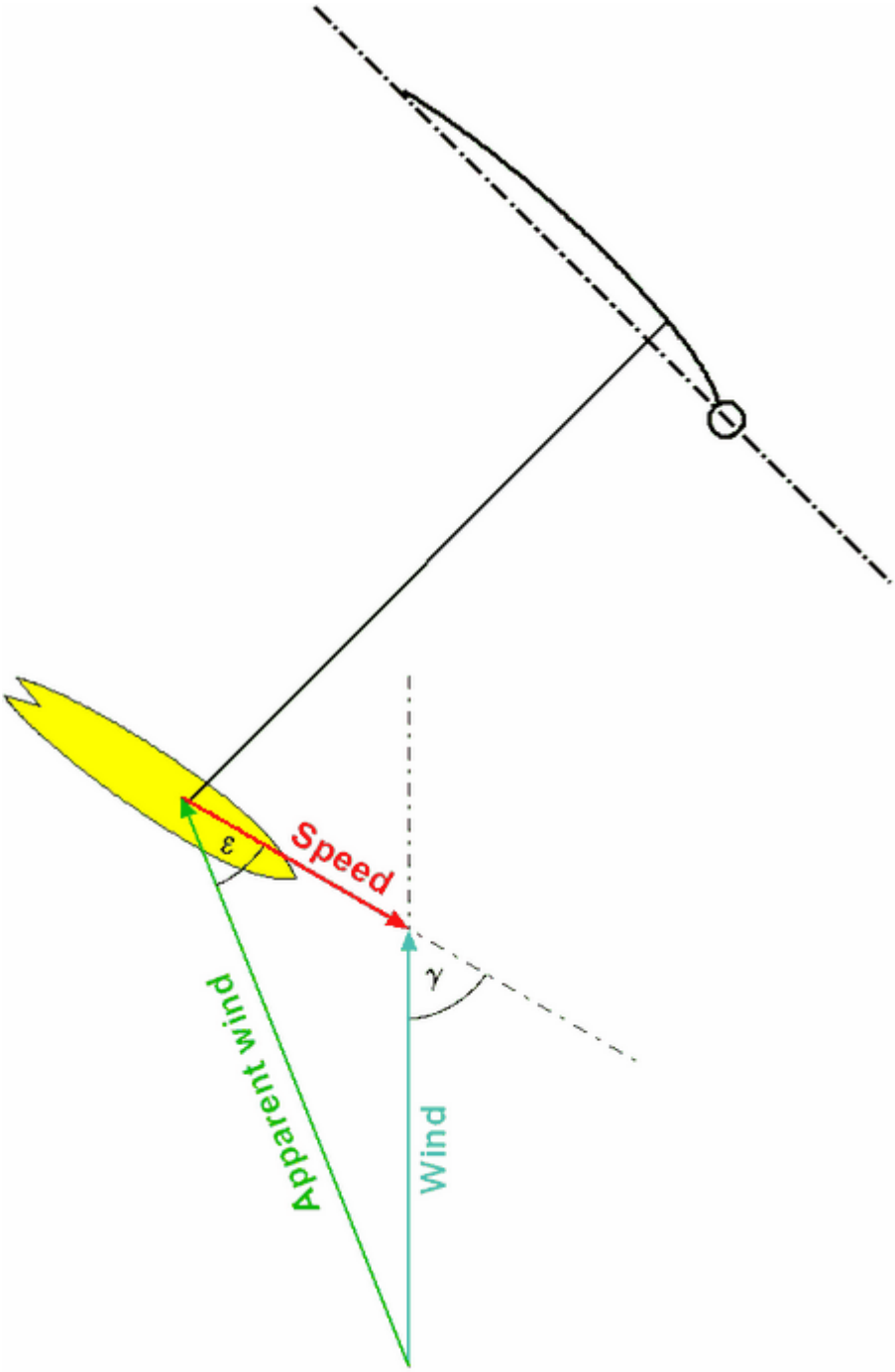
When using the term *course* we usually think in terms of a compass course as for example heading 135° when we go south east.
In the theory of sailing or kiting the compass course in not an issue but it´s quite important what course You go relative to the direction of the wind.
So polares of boats are commonly plotted over a heading or course relative to the wind.



Polare

A polare shows the complex reality of sailing. An angle of 0° would mean heading directly into the wind wich is impossible with a normal yacht or a kiteboard. 90° is Crosswind and 180° is going exactly downwind. Ths will be called the course here. We don´t care wheather the hot air balloon is drifting to the north, south, east or west but we can surely expect it to drift leeward. The angle between the fair wind and the Windspeed will be called γ. The angle between fair wind and apparent wind will be called

ε. So now we have a speed triangle with all the angles since the sum of the angles in a triangle is 180°. Hence the third angle of the speed triangle between wind and apparent wind is γ - ε.



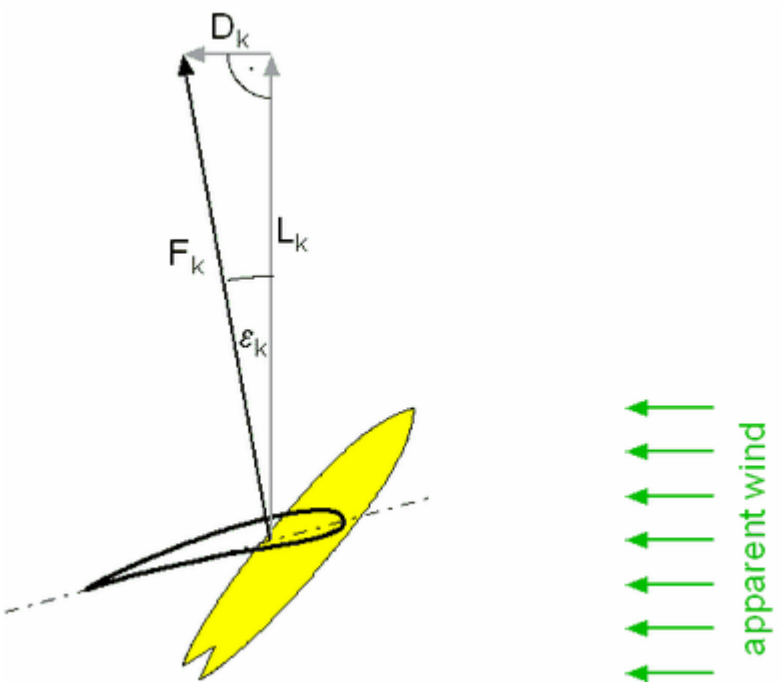
speed triangle with angles

This is the speed triangle that will be used here. It´s all there is needed to understand for the following discussion of forces.
You (the kiter) will always be in the edge with the speed and the apparent wind for these are what You actually feel. For the course and speed are seen relative to the wind rather than a compass course, speed and wind meet in the second edge of the speed triangle where the cours is to be seen as angle γ.

1.2 Aerodynamic and hydrodynamic forces

Any body held at a fixed position in a streaming fluid will have some *drag*. This force is oriented exactly in the direction of the flow, as this is the definition of *drag*:
Drag is the component of the force coursed by a streming fluid that is parallel to the flow.
As the force caused by a flowing fluid is not necessarily parallel to the flow, the component of the force perpendicular to the flow is called *lift*. The vectorial sum of both is the total fluiddynamic force.
The fluids in question for kitesurfing are air and water, so we have aerodynamics and hydrodynamics that are basically and principally the same in the low speed range that is covered by kitesurfing.

Aerodynamic forces:



Aerodynamic forces on a sail or kite

The kite can be seen as a wing. The centerline of the profiles have an angle of attac towards the airflow (apparent wind) and produce lift and drag. The drag itself can be distinguished to induced drag and parasitic drag but measured values only give one parameter that´s the overall drag. The lift of Your kite may be changed due to depower. Generally speaking You have a wing with a minimum lift coeficient and a maximum lift coeficient and drag so You have a polare. We seek to concentrate on maximum performance so that optimum L/D ratios are most interesting. Modern kites can have L/D ratios in the range between 5....8.
The drag causes the kite to fly at an angle further downwind than perpendicular to the wind speed. L/D is the cotangens of this glide angle:

L/D = cot(ε).

Glide angles of wings or kites can only lie in the range between 0° and 90°. So far for the theory!
The L/D ratios of modern kites correspond to glide angles between 7° for high performance kites and 12° for low performance kites. Using much of the depowe range not even having the trim right You are very likely using a kite with a glide angle between 12° and 20° every now and then even though You want it to perform at it´s best.
The Lift and Drag coefficients are more or less constant over a wide speed range and the lift and drag are described as:

$$L_k = \frac{\rho}{2} \cdot w^2 \cdot A \cdot c_l$$

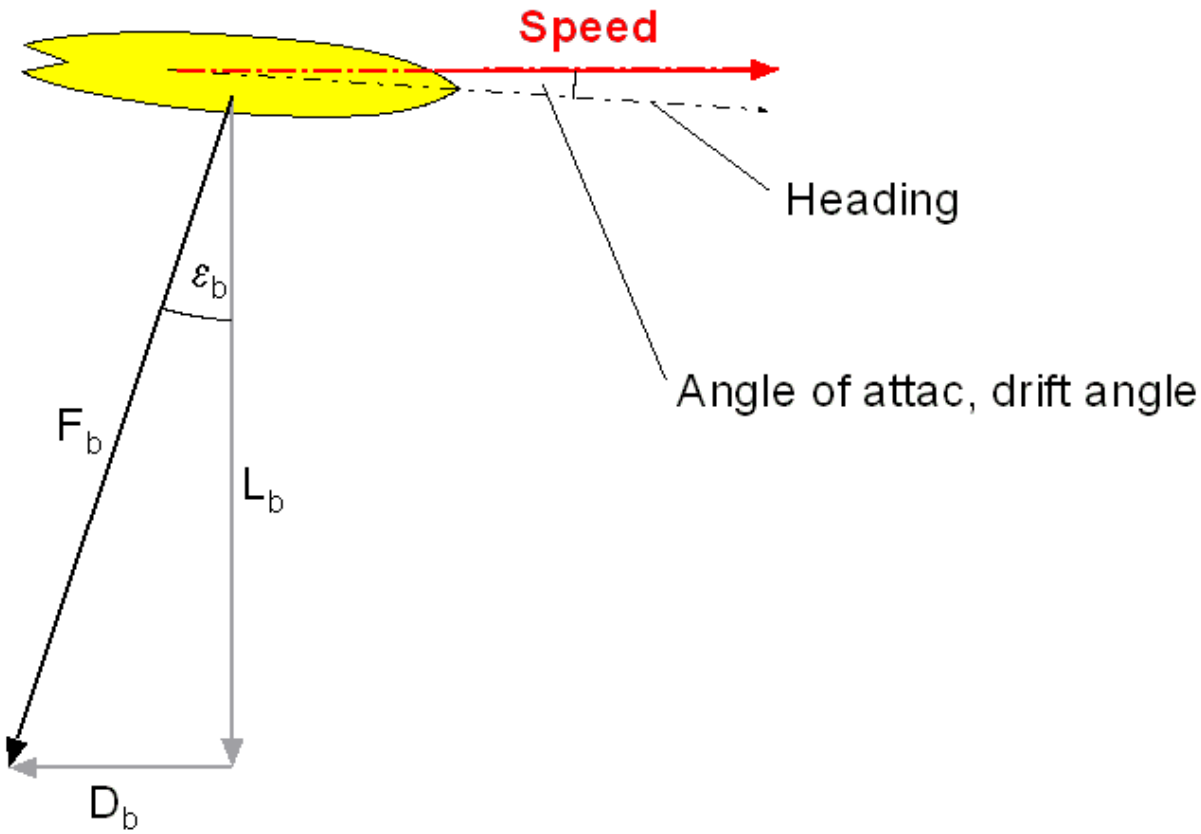
and

$$D_k = \frac{\rho}{2} \cdot w^2 \cdot A \cdot c_d$$

Aerodynamic lift and drag

Here ρ is the density [kg/m³] of the air, w [m/s] is the speed of the flow, here corresponding to the apparent wind. A [m²] is the flat area of the kite and c_l and c_d are the lift- and drag coefficients that can be seen as constant for each trim of the kite.
The most important thing we learn from these equations is that aerodynamic forces are proportional to the square of the apparent wind.
While there are measured L/D ratios for kites availabe, there is no public interest in lift or drag coefficients.
Typical values for maximum Lift coefficients of a kite lie in the range of 0.6...1.6 where a kite with a high maximum lift coefficient would be characterized as "grunty". C shaped kites tend to have lower lift coefficients just because they have not all of the area for producing lift. They tend to have somewhat lower drag coefficients as well as the "wasted" surface area works like a sort of winglet that allowes for a slightly sharper lift distribution over the wingspan. Some Years ago manufacturers used the projected area and those times the lift coefficients were more closely grouped together.
You are free to choose a slightly larger kite with a slightly smaller maximum Lift coefficient or vice versa to get the same lift.
The Range between maximum and minimum Lift coefficient is commonly referred to as depower ratio. Full depower kites have significantly lower minimum Lift coefficients while the maximum lift coefficients are dictated by the state of the art in developement and designers are not free to increase them as they would like to. Full depower kites and conventional kites have similar maximum lift coefficients. The difference in depower ratio comes corresponds to different minimum lift coefficients.

Hydrodynamic forces:



Hydrodynamic forces on a boat, keel, fins or edged kiteboard

Principally the hydrodynamic forces could principally be described the same way the aerodynamic forces are as well.

$$L_b = \frac{\rho}{2} \cdot w^2 \cdot A \cdot c_l$$

and

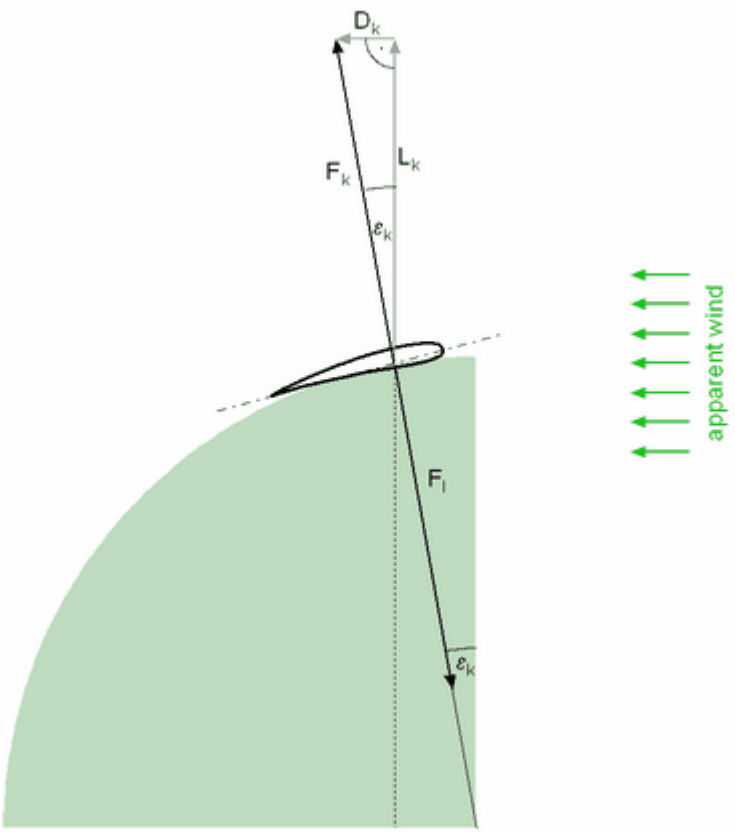
$$D_b = \frac{\rho}{2} \cdot w^2 \cdot A \cdot c_d$$

Hydrodynamic lift and drag

This is not too common though, because in sailing especially the more interesting Yachts as speed sailing is concerned have vastly changing lift- and drag coefficients over speed and where the parameters change vastly the idea of having constant coefficients is not of much help. The behaviour of boats is rather described using polares than simple coefficients and the performance of a kiteboard is very much dependent on riding style, rider skills and chop or waves. So there is no such thing as a polare of a kiteboard. There is no such thing as lift and drag coefficients either so most everything that is known about a kiteboards efficiency must be deduced from speed performance taking into account the kite used, the rider and the conditions of the speed run.
All there is is the fact, that all glide angles ε_b must lie in the range between 0° and 90° as this is the theoretical frame for all hydrodynamic performance.
The hull of a boat or the kiteboard are usually driven with an angle of attac so that keels, fins or the edge of Your kiteboard can produce a force component perpendicular to the speed. The angle between the heading of the centerline and the speed through the water is usually referred to as drift angle in sailing theory but we will call it angle of attac.
Just like You can stall a wing or sail You can stall a keel or fins as well ba goint too slow with too much a force perpendicular to the heading. Watch Your kiteboard and You may find Youself riding with angles of attac up to ~ 20° especially as You try to go gain hight close hauled.

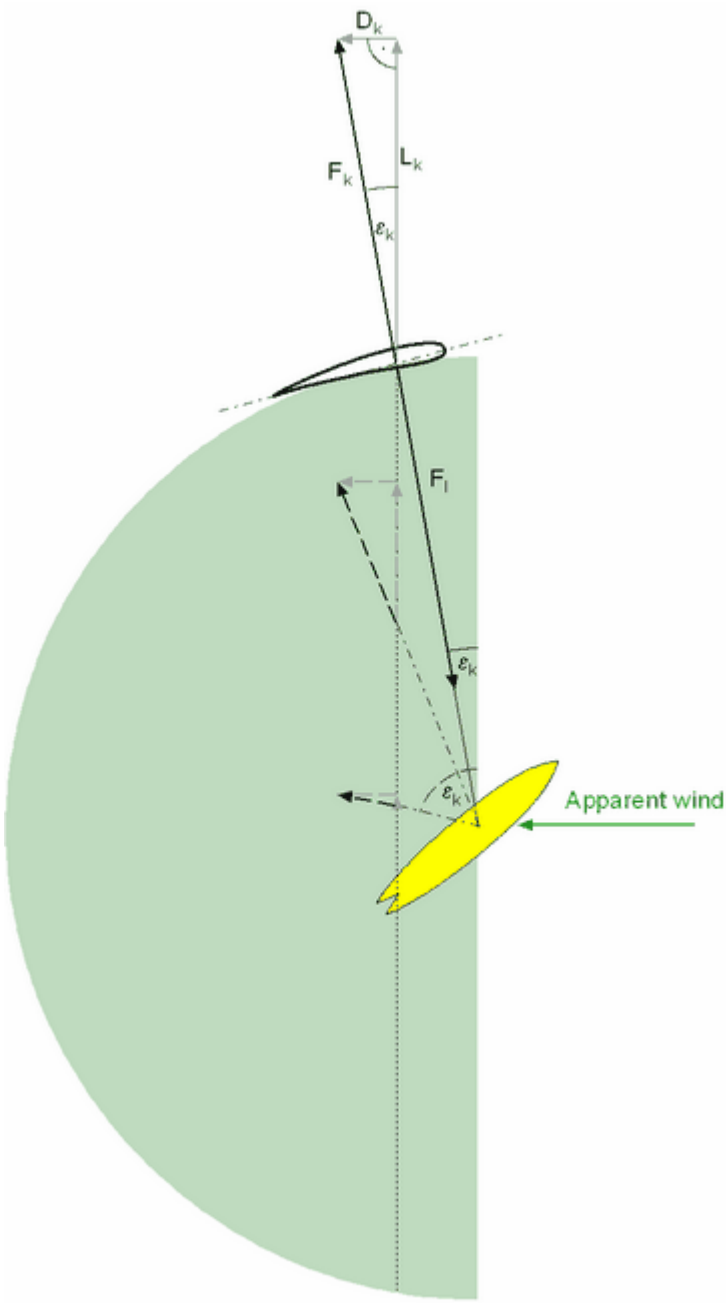
1.3 The wind window

This is specific to kite flying as there is no wind window for wings os sails. The kite is connected to You through the flying lines and therefore can only fly around You at this distance. The wind window has the shape of 1/4 of a sphere.



Wind window seen from the side with kite in the zenith

In the side view of the kite and wind window You can see, that as the kite is swept back by the aerodynamic drag. All positions of the kite at it´s edge of the wind window are seen on the dotted line. In steady state the kite can not fly through the powerzone but will always have balanced forces at the edge of the window. Medium- and power zone are only accessible by dynamic steering of the kite. The Force You need to hold the kite with the kitelines is antiparallel to the aerodynamic force and of the same strength.



Wind window seen from above

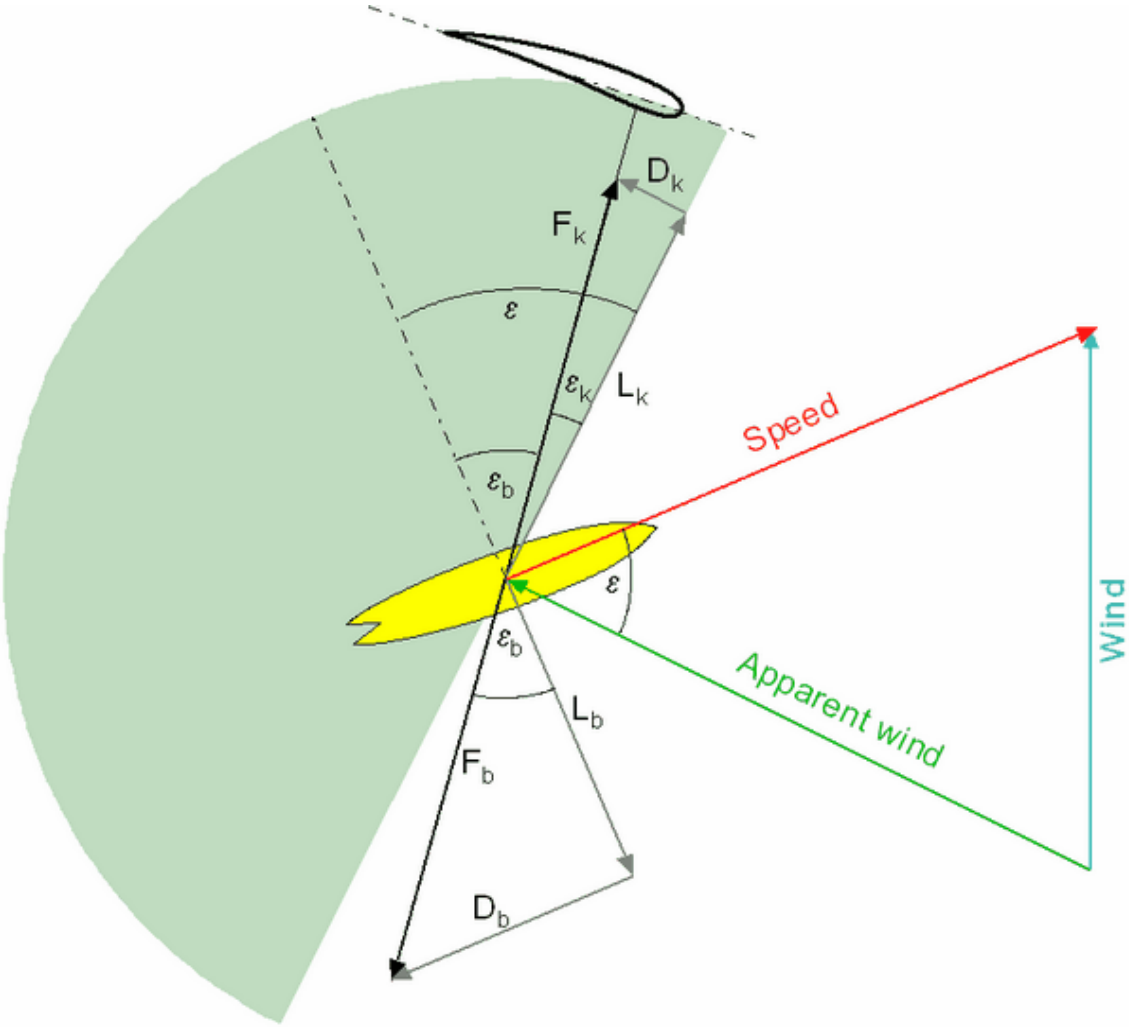
It is most important to understand that the horizontal and vertical forces have to be balanced while steady state kitesurfing or sailing.
The vertical components are usually taken as balanced because the hull of a boat provides the amount of buoyancy and the kiteboard glides on the waters surface. The horizontal components must be balanced and here only the appropriate components may be taken into account. It is seen that while the drag of the kite is always there to slow you down the horizontal component of the lift depends much on where You steer the kite. Close to the zenith the performance is much weaker than with the kite flown low. That´s why You´ll never see a speed run with the kite close to the zenith. On the other hand it´s very common and usual to fly the kite high every now and then and have an aerodynamic performance much lower than the L/D of the kite would allow for. It´s a means of controlling the ride and reducing the efficiency of the kite is needed the most with 2 line kites or limited depower.
In all of our further discussion we will see the glide angle as seen in this picture i.e. the horizontal glide angle. But we may keep in mind, that the vertical component of the kite´s lift influences the

performance of our kite boards. The board might perform significantly better with the kite carrying most of Your weight.
Another very important fact is that the wind window is positioned with respect to the apparent wind no matter what and where the true wind might be. The forces on the kite and therefore the wind window are caused by the airflow around the kite and that´s the apparent wind.
That said it´s now time to bring these facts together and enter kitesurfing theory.

1.4 Kitesurfing theory

1.4.1 Balance of forces

In steady state all of the external forces on a have to be balanced. The sum of all forces is 0.
This is Newton´s 3rd law: "actio = reactio".
If the external forces are not balanced the mass of Your body is accelerated in the direction of the external force. That´s what we experience during a wipeout or jump but for now our task is to catch that hot air balloon!

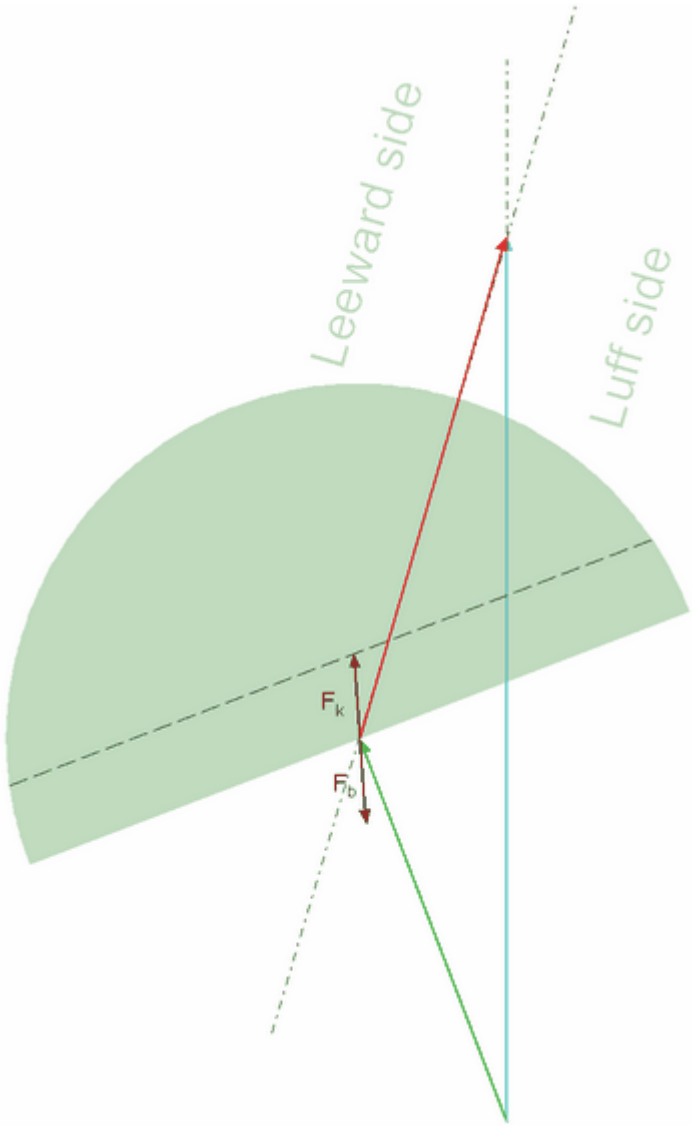


Speed triangle and forces

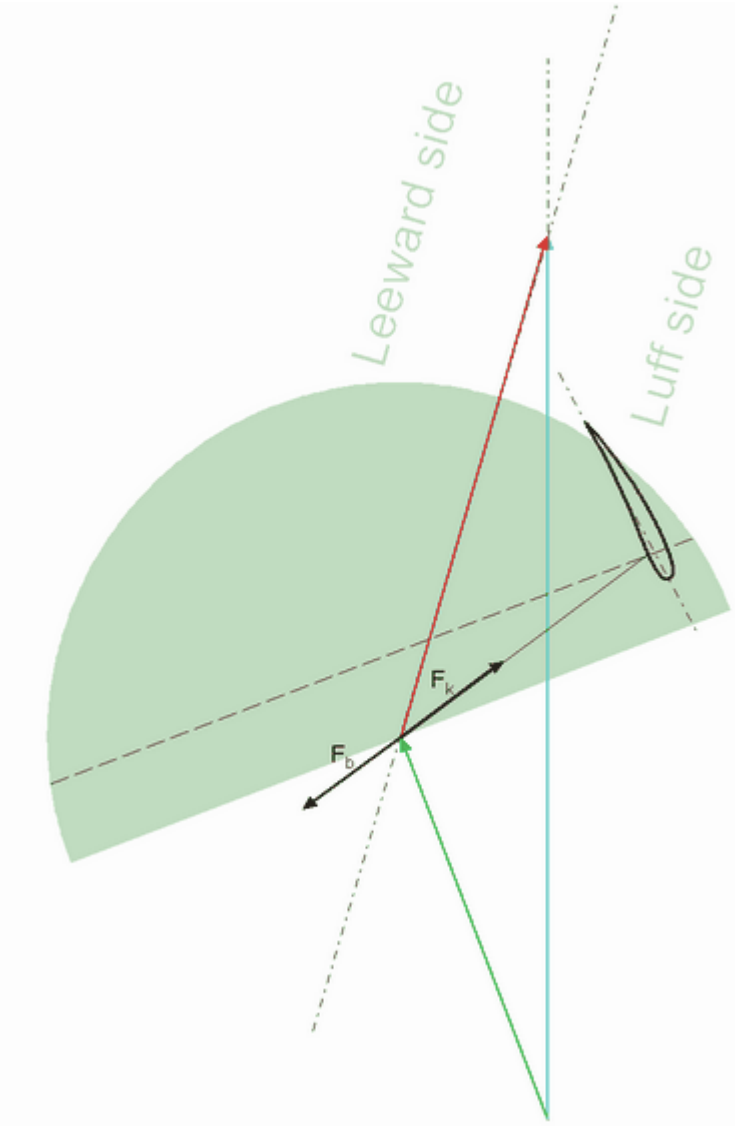
That´s what it looks like in a steady state. The aerodynamic foce and the hydrodynamic force are antiparralel and equal in size. Thus You can measure the glide angle of the board between the dotted line perpendicular to the speed and the aerodynamic force as well.
The sum of ε_k and ε_b is ε and this must be the angle between apparent wind and speed as well. You can refer to this angle as the overall glide angle or glide angle of the craft. For is the sum of two real glide angles, each in the range between 0° and 90° the overall glide angle ε may be in the range between 0° and 180°.
The angle between apparent wind and speed is the overall glide angle ε.
The amount of force You need to pull Your board forward naturally depends on riding skills, board size and type, chop, Your weight and the vertical component of the kite pull. The kite must provide the same horizontal force so it´s a question of depower and a properly chosen kite size to make these forces match. If the horizontal forces don´t add to 0 You accelerate. A brief description of what tha leads to is given in chapter "Imbalance of forces".

1.4.2 Two ways of low efficient kiting

While in the last picture the glide angle was well below 90° - we will call this the high performance part of kite theory - You are not free to choose the side of the wind window where to fly the kite. With low performance there is a second kind of kiting as well...



"Normal" kiting or propelling

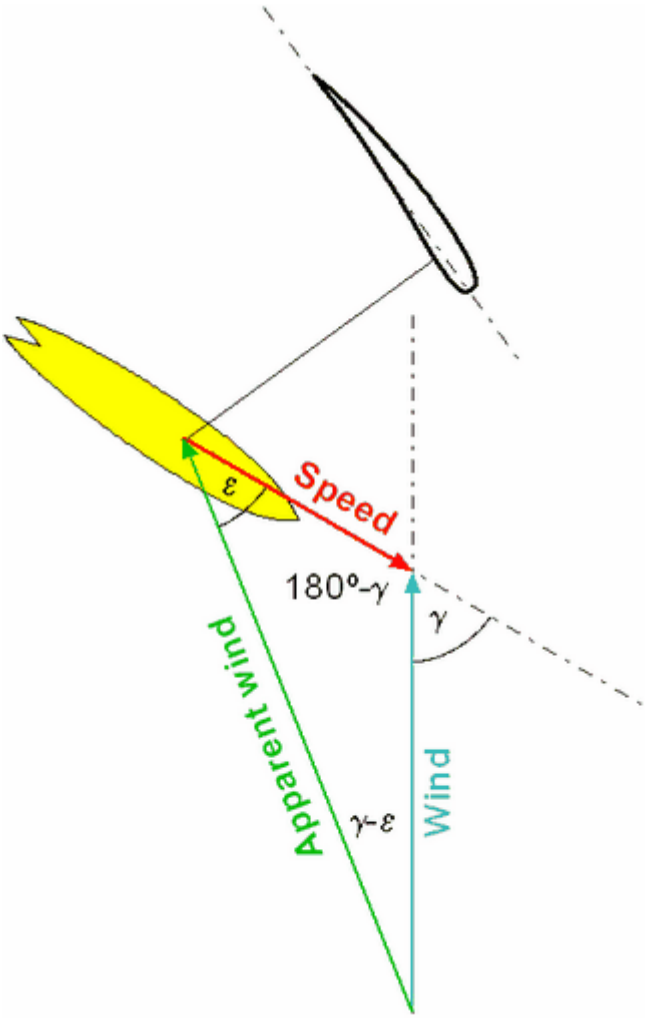


"Strange" low performance kiting or windmilling

For wind and apparent wind are always on the same side of the course You go, this side of the course is called the luff side. The other side is called the leeward side. Most of the times and especially with high performance You have the Kite in the leeward side. This is normal sailing and normal kiting and normal kiting is called "propelling" in this manual. To understand why it´s called "propelling" You might have to refer to the analogies chapters: *Threaded shaft* and *Prop missile*.
If hou have the kite on the luff side of Your course and lean towards the leeward side that would normally be Your toeside, this is called "windmilling" here. There´s a completely different kitesurfing theory needed for windmilling as the angle ε is not the sum of ε_k and ε_b here.
This whole steady state theory is focused only on propelling. For Windmilling it can be shown that 180°- ε = ε_k - ε_b.
A highly dynamic way of windmilling can be used to slow down after a speed run though: Ster Your kite back to the other side of the wind window and use it as a parachute for slowing down. It feels sort of strange but sometimes it´s quite helpful, too.
Do we need a theory of windmilling with Your kite? As there is no steady state windmilling with high performance we chase the balloon propelling.

1.4.3 Speed triangle

It´s time to summarize some aspects and look at the sped triangle as we might use it for calculations.



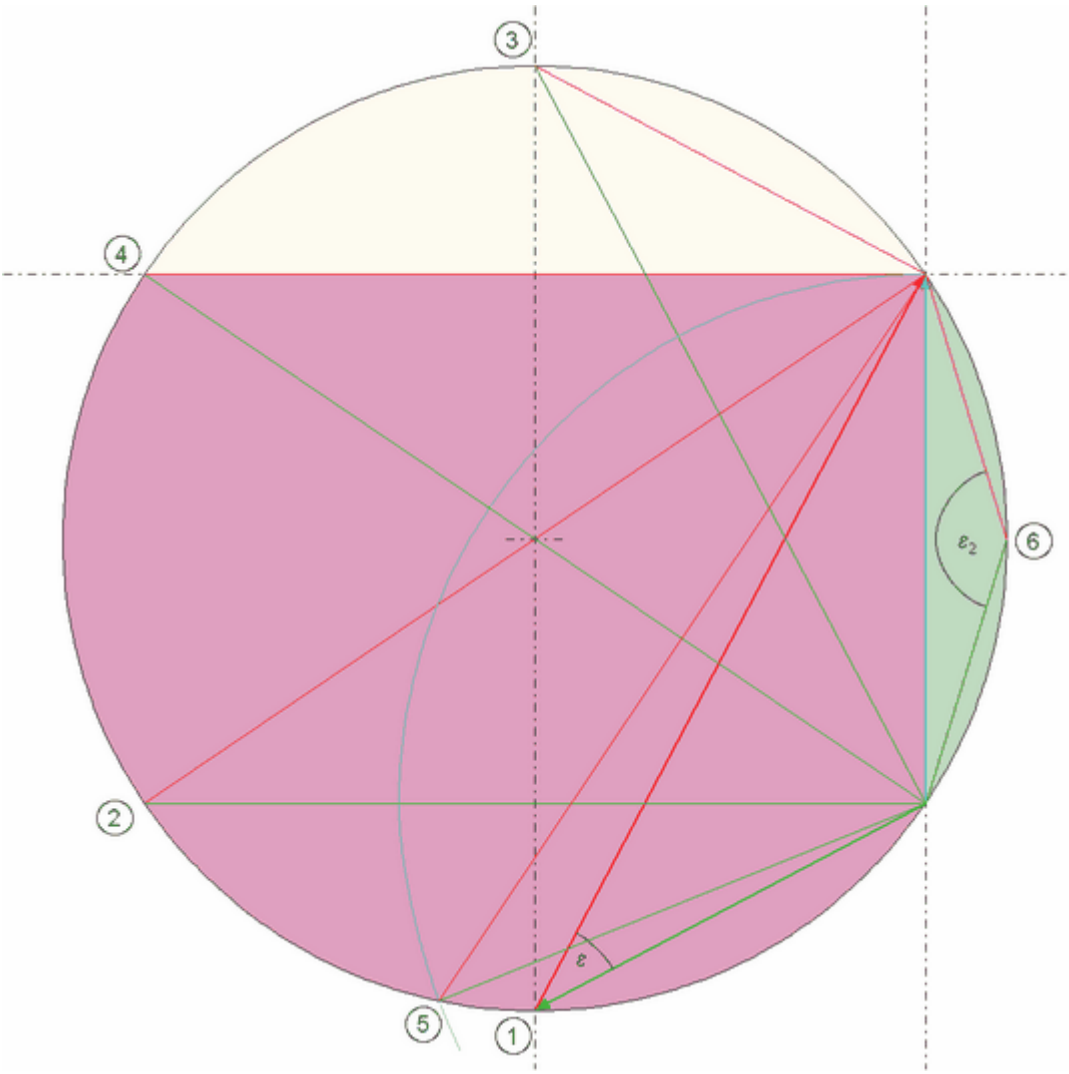
Speed triangle for practical calculations

For calculations it is helpful to use the law of sines:

speed / *sin*(γ-ε) = apparent wind / *sin*(180°-γ) = wind / *sin*(ε)
For further convenience You can use *sin*(180°-α) = *sin*(α) so apparent wind / *sin*(180°-γ) = apparent wind / *sin*(γ)
The plot of parameters below has been calculated this way.

1.4.4 Performance circle

One specific glide angle ε given, all possible speed triangles for kiting at a steady state have the same angle ε between course and apparent wind. It can be shown, that all these speed triangles have one common circumcircle.
There are two glide angles sharing the same performance circle with only only one exeption: ε = ε_2 = 90°. One of them beeing lower than 90° and one beeing larger than 90° so they are called high efficiency glide angle ε and low efficiency glide angle ε_2. These two are related so that ε_2 = 180° - ε. The mathematical background for all this including helpful hints and full proof for every single detail is given in the Math appendix.
One fact that can be seen at a glance at the performance cycle is, that with the low performance ε_2 > 90° it is not possible to gain hight, ride faster than the wind or have an apparent wind higher than windspeed.



Performance circle and points of special interest

The performance circle will presented here as follows:
The Wind is a vertical vector pointing upward. The wind vector will always be on the right side of the circle and as the wind vector is one side of the speed triangle both ends lie on the circumcircle as well as the third edge of any speed triangle with the same glide angle ε.
There might be something like a optimum performance performance circle of Your craft. Concider ε_opt the best possible glide angle of Your craft and every possible steady state with every glide angle ε higher than ε_opt will lie inside the optimum performance circle. The yellow part of the circle

is where You gain hight, the red and green part is where You loose hight.

We want to catch a hot air balloon so we´ll very much concentrate on the red high performance part. For some poins of special interest it´s easy to see, where on the circle they will have to be.
The fastest possible speed must be a diameter of the performance circle as well as the highest possible amount of apparent wind. Thus, both must have the same value. The best VMG upwind must be achieved at the topmost point of the circle while the best VMG downwind can be found at the downmost point opposite of the best VMG upwind.
The best VMG downwind is exactly 1 times the wind speed higher than the best VMG upwind.

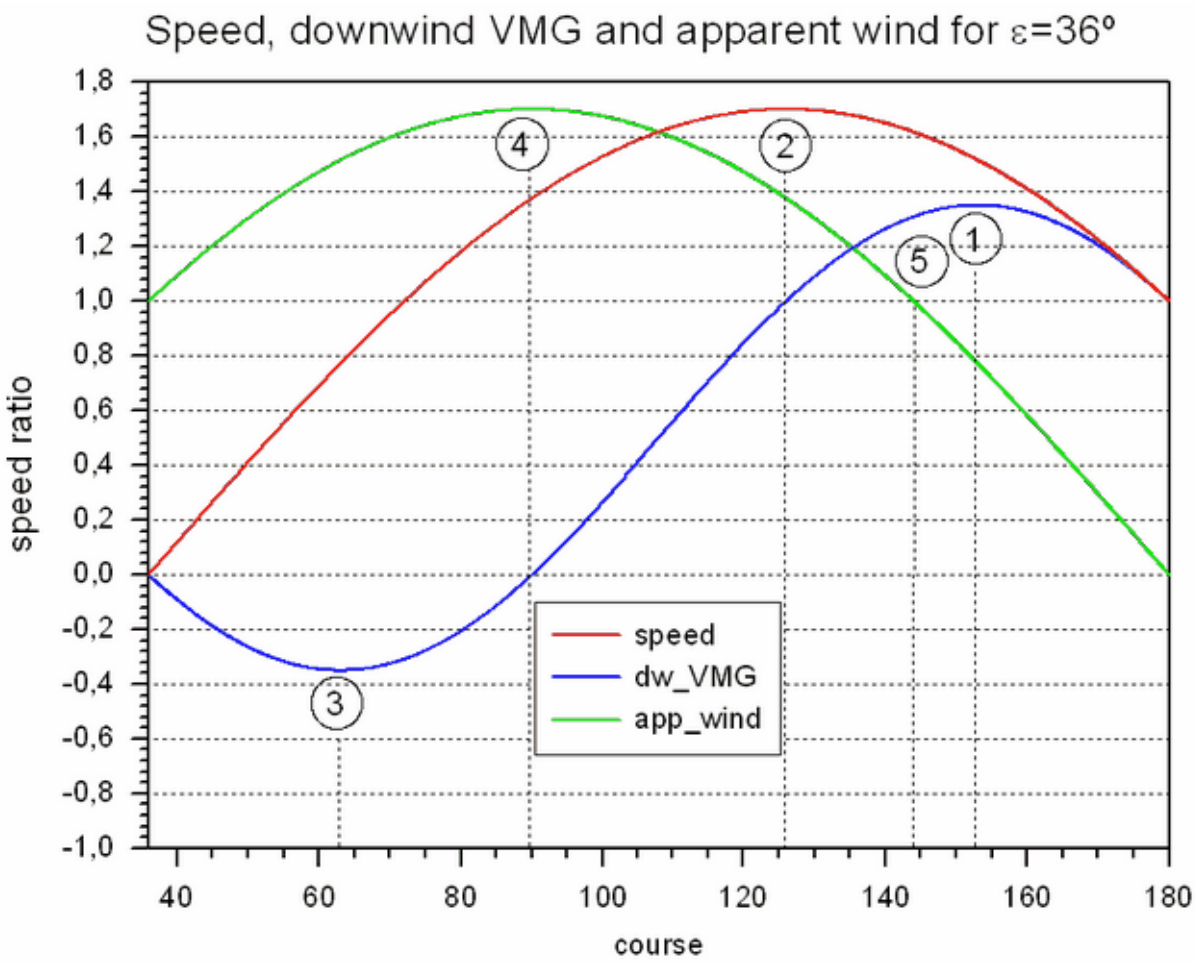
Here´s a complete overwiev:

POI $\varepsilon < 90°$	special interest	speed ratio s	apparent wind ratio v_app	course γ	downwind VMG ratio VMG_dw
1	max. downwind VMG	$\sin(135°-\varepsilon/2) / \sin(\varepsilon)$	$\sin(45°-\varepsilon/2) / \sin(\varepsilon)$	$135° + \varepsilon/2$	$(1/\sin(\varepsilon) + 1) / 2$
2	max. speed	$1 / \sin(\varepsilon)$	$1 / \tan(\varepsilon)$	$90° + \varepsilon$	1
3	max. upwwind VMG	$\sin(45°-\varepsilon/2) / \sin(\varepsilon)$	$\sin(135°-\varepsilon/2) / \sin(\varepsilon)$	$45° + \varepsilon/2$	$- (1/\sin(\varepsilon) - 1) / 2$
4	max. v_app	$1 / \tan(\varepsilon)$	$1 / \sin(\varepsilon)$	90°	0
5	v_app = 1	$2 \cos(\varepsilon)$	1	$180° - \varepsilon$	$1 + \sin(90° - 2\varepsilon)$
6	corresponding low performance $\varepsilon_2 = 180° - \varepsilon$	$s < 1$	v_app < 1	$\varepsilon_2 < \gamma$ and $\gamma < 180°$	0 to 1

Points of special interest for steady state kitesurfing given the glide angle ε

1.4.5 Parameter studies

With these expressions many calculations can be performed so this is where the performance circle is really helpful.
Here´s an example given for ε = 36° that can be achieved with a good production kiteboard and a not too low performing kite.



Plot of some parameters for ε = 36°

The points of special interest are shown and here are the values:

POI ε = 36°	special interest	speed ratio s	apparent wind ratio v_app	course γ	downwind VMG ratio VMG_dw
3	max. upwwind VMG	sin(45°-36°/2) / sin(36°) ~ 0.7724	sin(135°-36°/2) / sin(36°) ~ 1.5159	45° + 36°/2 = 63°	(1/sin(36°) - 1) / 2 ~ - 0.3507
4	max. v_app	1 / tan(36°) ~ 1.3764	1 / sin(36°) ~ 1.7013	90°	0
2	max. speed	~ 1.7013	~ 1.3764	126°	1
5	v_app = 1	~ 1.6180	1	144°	~ 1.3090
1	max. downwind VMG	~ 1.5159	~ 0.7724	153°	~ 1.3507

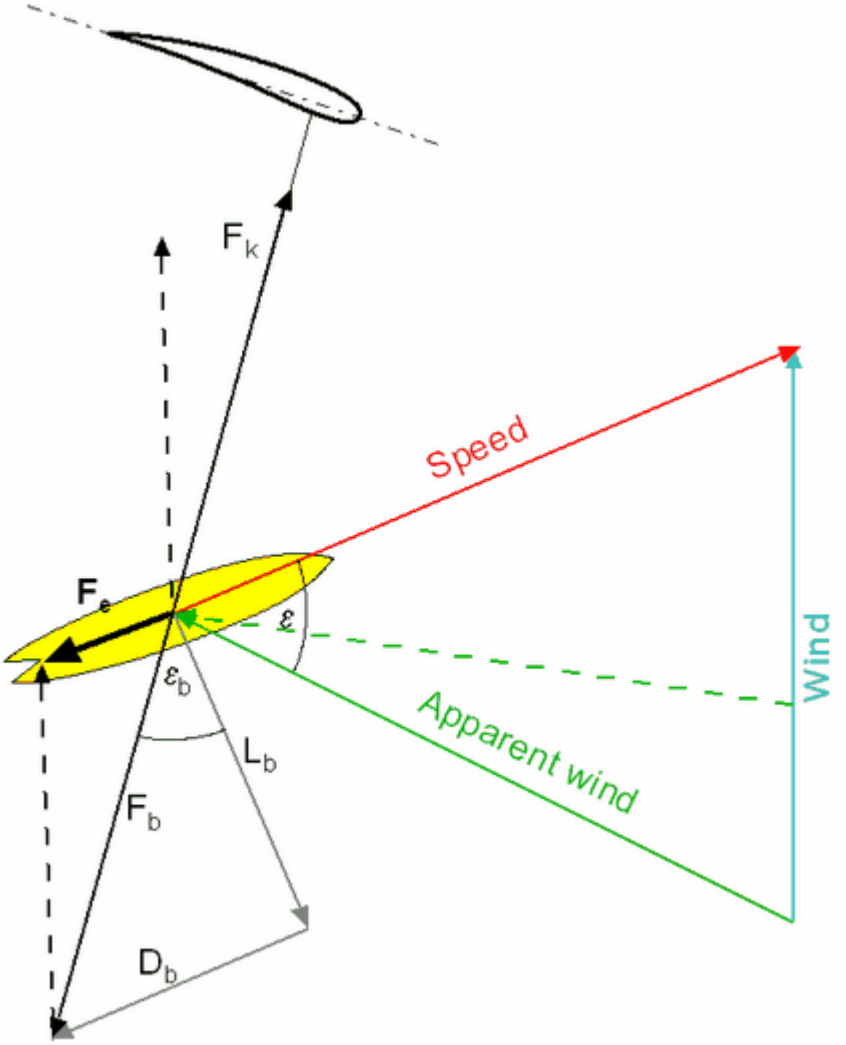
Points of special interest for ε = 36°

What You can make of it depends on Your riding skills, feeling and equipement a lot. But it´s quite promising and interesting to see, that at a course of 144° downwind You feel the apparent wind of exactly the wind speed so You should not be under- or overpowered while You race downwind with a downwind VMG of 1.31 times the wind speed. It should be fairly easy to catch that balloon even with an ordinary 2 line kite!
Should You be in a hurry it´d be advisable to go well powered to moderately overpowered because at a course of 153° You can have the maximum downwind VMG achieving some 1.35 times wind speed but You´ll only feel 77% of the wind speed, the apparent wind. So with the dependancy of Lift to the square of the apparent wind that´d be only 60% of the lift You had while lunching the kite.
One more thing can be seen in the plot. It depends on the depower ratio of Your kite how much apparent wind You can handle. That in mind one might plot the ratio that represents kite lift: the square of the apparent wind ratio. With a maximum apparent wind ratio of 1.7 You´d have to hold 2.9 times the lift You had during relaunch so this might only be handled if You go out way underpowered on a large board or use a full depower kite.

1.5 Dynamic aspects

1.5.1 Imbalance of forces

If the external forces don´t match You´ll be accelerated and (by definition of the term "steady state") leave the steady state. The overall external force F_e i.e. the vectorial sum od aero- and hydrodynamic forces is mass times acceleration.

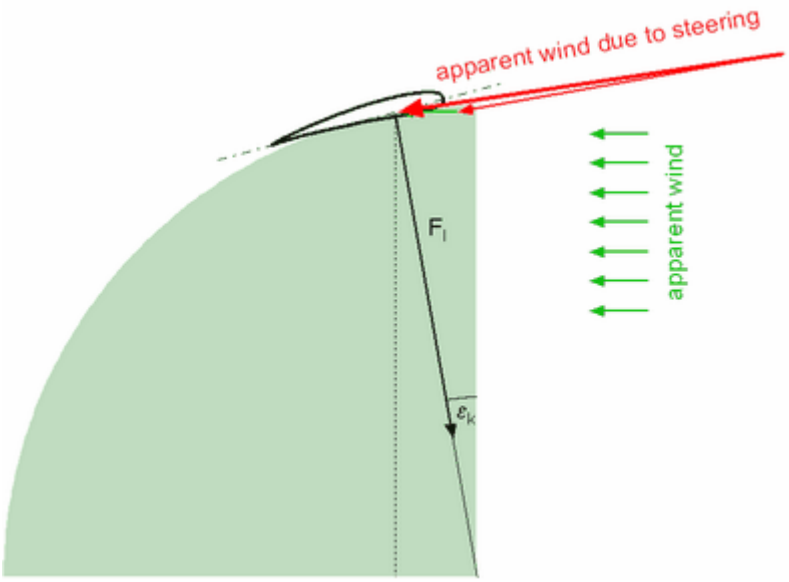


Kiting in a lull: External force F_e

The resulting external force F_e in this example will slow You down. This leads to decreasing speed and the resulting change in apparent wind, forces, speed and so on...
Dynamic kiting is the most of a usual thing but there´s no special dynamic theory given.
You can analyze the aerodynamic and hydrodynamic forces, the influence on speed vectors that results of the forces given and continue analyzing the next speed triangle.

1.5.2 Dynamic kite steering

The major difference between kitesurfing and windsurfing is that you can steer the kite dynamically and fly it through the wind window. As we have seen in the chapter *Aerodynamic and hydrodynamic forces* the lift will increase with the square of the apparent wind of the kite. For a kite is able to reach a speed of approximately L/D times the wind speed this would result in 50fold lift for a high performance kite with nochange of the lift coefficient. Dynamic steering of the kite greatly influences the aerodynamic forces but factor 50 would be lifethreatening! Fortunately there´s an effect that results in lower forces than that. As we coud see in the wind window seen from the side the kite is swept back due to it´s aerodynamic drag. If the kite itself creates apparent wind this apparent wind will not contribute to lift in the same way as an increas in windspeed would do.



Change of the angle of attac due to dynamic steering

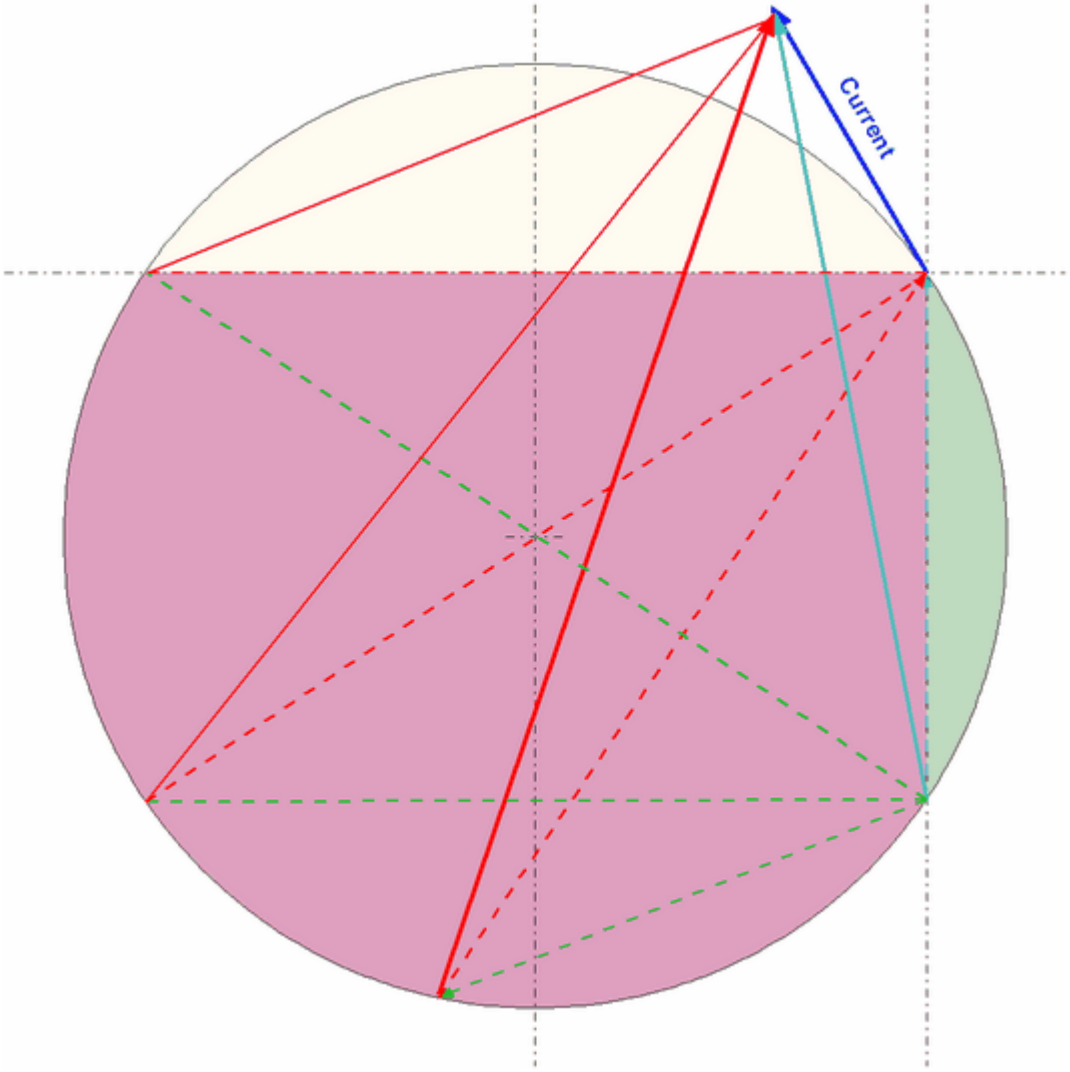
Kite design has to take drag and L/D into account, so the designed angle of attac is reduced by ε_k compared to a kite without drag.
Thus the additional appparent wind created by kitesteering hits with a lower angle of attac what helps reducing the additional lift due to kitesteering.
High performance kites will not only fly faster than intermediate kites, the change in angle of attac is smaller also because ε_k ist lower with a high performing kite. That´s what makes high performance kites more dynamic and - in the upper wind range with smaller sizes - more dangerous as well.

2 Analogies

The kitesurfing basics should be good enough to catch that hot air balloon with a kiteboard and kite.
Now there is a theory how to kite what courses and speeds once the efficiency of kite and board are given, one might think about related problems and how to apply this theory to them or at least to get a clue what these are about.
One of the more important questions - of course! - is:
"How to launch a hot air balloon, kite downwind and catch that balloon again in a current?"
The kitesurfing theory did not take into account what will happen, if there is a current against the wind, current with the wind and the like.
The Threaded shaft chapter discusses propellers using the wing theory that will only apply if the propeller is shaped unusual enough to fulfill the assumptions the wing theory is based on.
The Prop missile chapter actually discusses propeller driven craft making use of the propeller theory - a completely diffenent way of thinking that applies to propellers that look more usual.

2.1 Kiting in a current

Why not say a few words about "Wind relative to the water", applying the sailing theory and add the speed of a current to all of the speeds?
So far we have assumed the water beeing motionless so You would feel and measure the same wind speed on the beach and on a raft. Now what happens in a current?



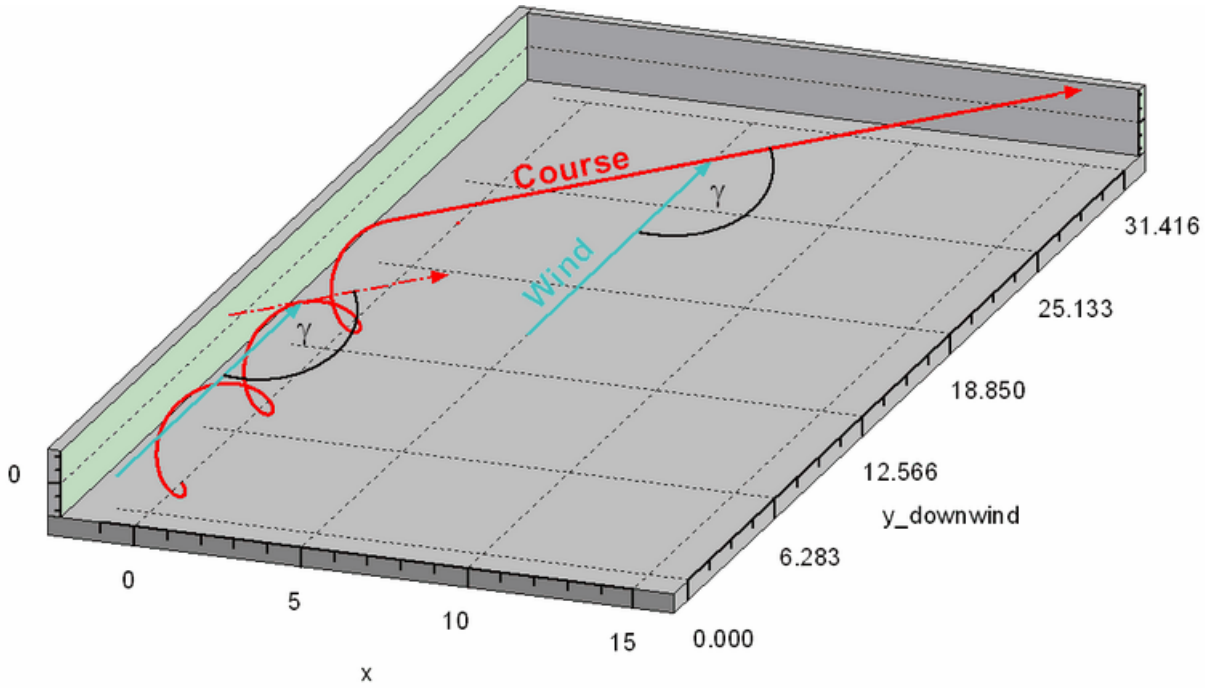
Steady state kiting in a current

Here we have two coordinate systems in one picture: ground fixed and water fixed.
May it be unusual but i call the wind relative to the water "wind" as it has been done all through this manual and i call the wind over ground. The water ficed coordinate system is giveb dotted and the grond fixed system is shown with solid lines.
That´s the procedure:
Subtract the current from the wind over ground vectorially to get the wind over water. Now You are in the water fixed coordinate system and use the steady state theory as if there were no current.
After having found the point of special interest You know course and speed over water so all there is to do is vectorial adding the current to the speed. That gives the speed over ground.

An astonishing case for doing this transformation is kiting in a current when there is no wind over ground.
As You can kite downwind faster than the wind it is possible to kite with a VMG against the current if Your board and kite are high efficient and the current is strong enough. But this is a bit of academic because in real life a current at let´s say 15 knots comes seldom without rapids and whirlpools.

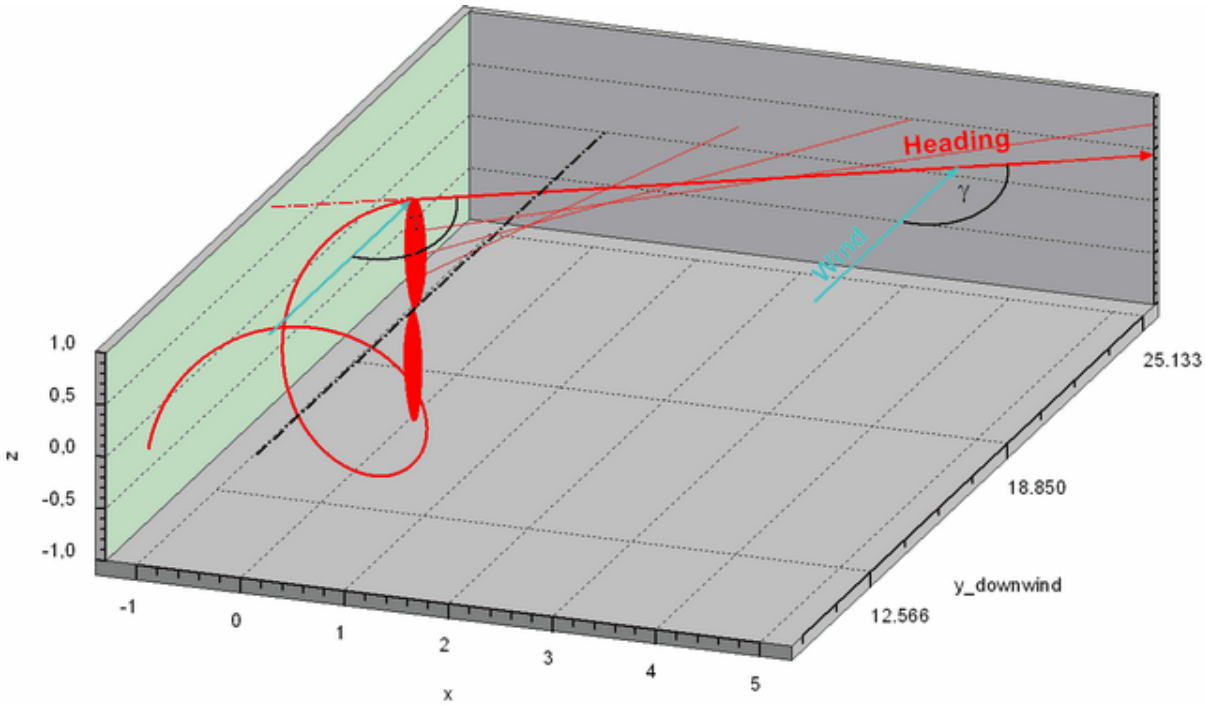
2.2 Threaded shaft

Now imgaine there was a threaded shaft and a rotor moving along it´s axis so with every movement parallel the shaft´s axis the rotor would have to rotate according to the threaded shaft´s pitch.



threaded shaft kinematics

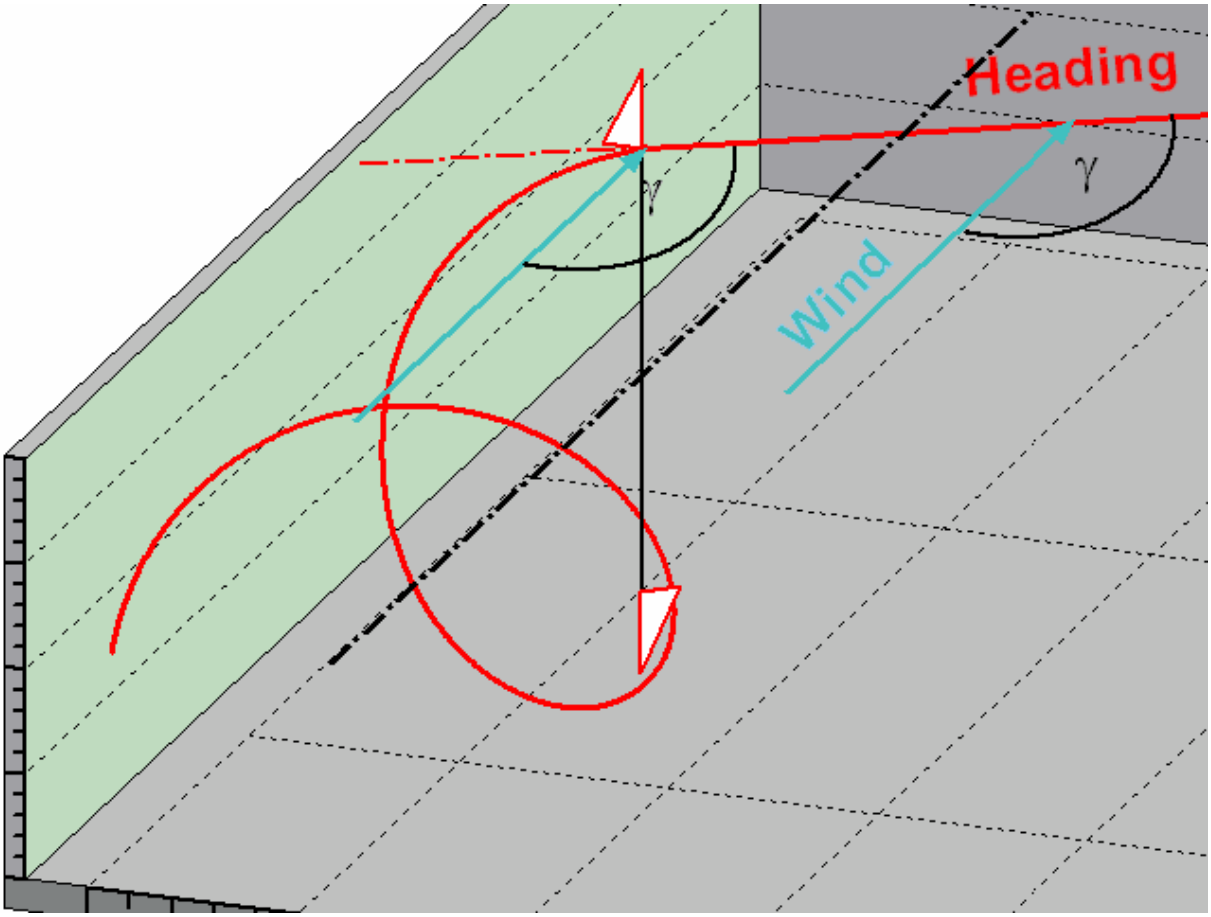
If we had to think about a usual propeller instead of a sail, wo would have to deal with some problems that show it is not at all clear, that the wing theory applies to the blades of a propeller.



propeller and wing theory

While all of a sail goes at one course, the radial sections of a typical propeller don´t.

The outermost section of a propeller has the largest crosswind speed component while the innermost section goes downwind. The blade elements of a propeller go all kinds of courses at the same time. One way to deal with this are rules of thumb for typical propellers. One of them says that the blade element at 75% of the radius of the propeller would be representative for the overall behaviour. The thought behind this is that there is a center of effort of the forces on each blade of the propeller, that would be representative for the whole propeller´s bhaviour.
These rules of thumb are commonly used but however this is getting some kind of shirt-sleeved so one might think of another solution that can be understood using wing theory.

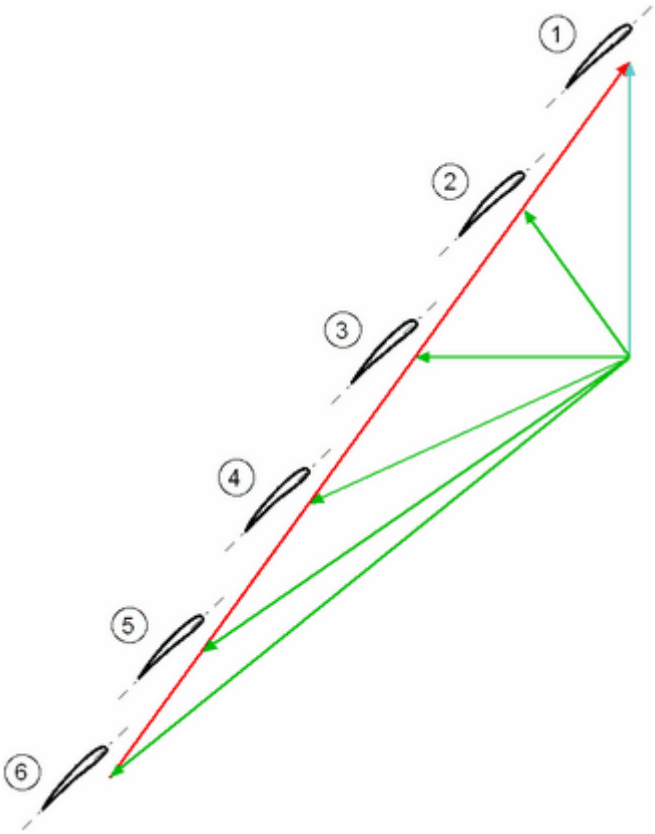


propeller design for wing theory

If all of the aerodynamically active parts of a propeller are concentrated around one radius, the prop looks much like a sail and all active parts of the propeller go very much the same course. For such a rotor it makes sense to use wing theory rather than propeller theory.

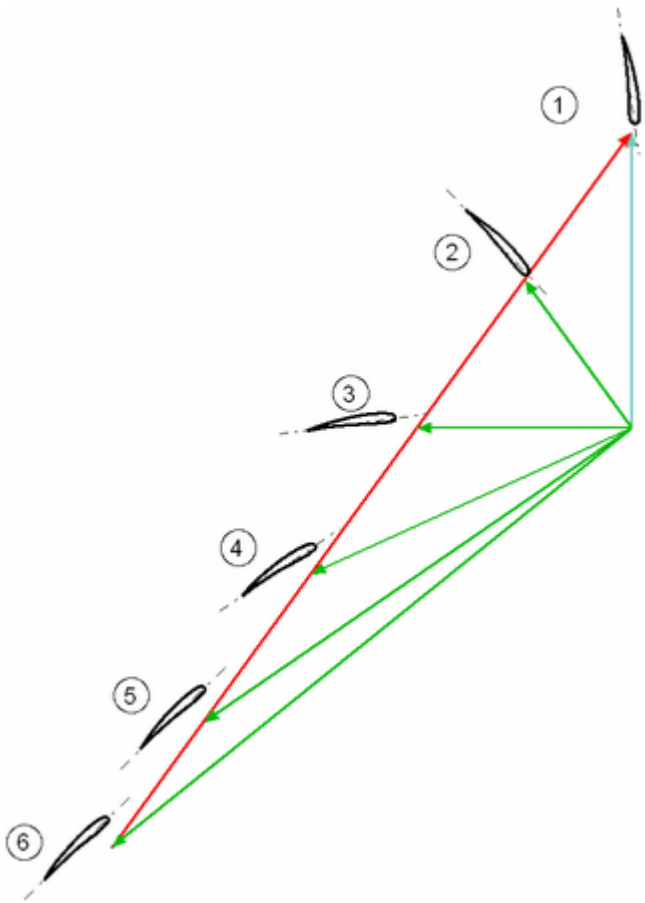
That said we choose a kite like propeller with a glide angle of 8° and let´s say we can build a threaded shaft with a glide angle of 10° so the glide angle of the craft will be 18°. The optimum course for downwind speed will then be 135° + 1/2 x 18° or 144°.
So we already know how it´d look like to be at steady state but we might have to get there in the first place?

The rules are, that the threaded shaft points dead downwind so the course of the active parts of the "kite propeller" is fixed to 144°.
Speed triangles will be wind speed plus all the speeds between 0 and maximum speed, where it comes to a steady state that is part of the performance circle.



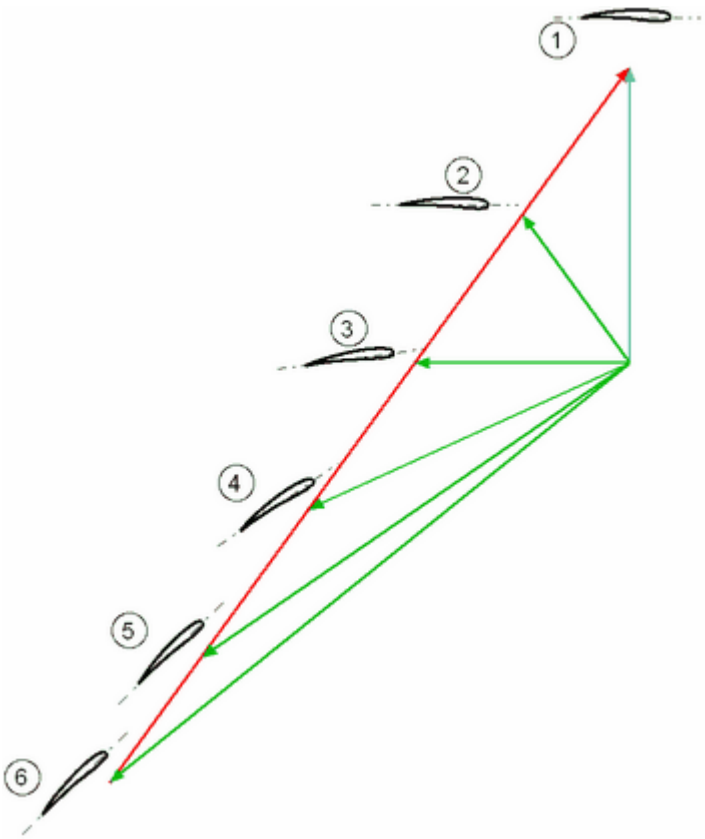
Acceleration of a record downwind craft with fixed pitch propeller

Here´s a scetch of what would happen to a fixed pitch propeller on a threaded shaft.
Point 6 is on the performance circle and would be design speed. The angle of attac of the propeller´s blade wouls fit just fine here for it´s the optimum angle of attac.
The same thing does obviously not work properly at standstill. Point 1 shows that the blade might even create some lift to the left that prevents the vehicle from accelerating. At point 2 (downwind VMG = wind speed / 2) things are not much better. The profile will stall and it´s not at all clear, wheather the resulting aerodynamic force would accelerate the craft. The same poor performance at point 3 (downwind VMG = wind speed) acceleration or not? This will depend on the profiles polare at very high angles of attac.
At point 4 it seems safe to say it´ll work. But it would still have a lousy performance.
At point 5 and 6 the angle of attac looks great so this will work. We conclude from this that such a craft would accelerate to design speed if only it would make it to a state close enough to optimum performance.
So what to do? At low wind speeds You might use the propeller as a windmill:



Acceleration of a record downwind craft windmilling

Here we have a variable pitch propeller so it can be used just like a windmill. Again, at 1 we are standing still and the angle of attac is chosen so that the lift as well as the drag will contribute to accelerating the prop.
At downwind VMG=1/2 wind speed (point 2) still both components accelerate the propeller. At points 3 to 5 the propeller will stil accelerate because it´s pitched to optimum angle of attac and the angle ε is larger than at point 6 that is on the performance cycle with ε_craft. In the beginning of the process the aerodynamic force is directed to the luff side of the course so this is precisely what happens in a windmill. Later on, from oints 3 to 6 the aerodynamic force is directed towards the leeward side and that is called propelling here. Whay? That´s because if the propeller was mounted on a drivetrain rather than a threaded shaft that drivetrain would have to be driven like the propeller of a plane.



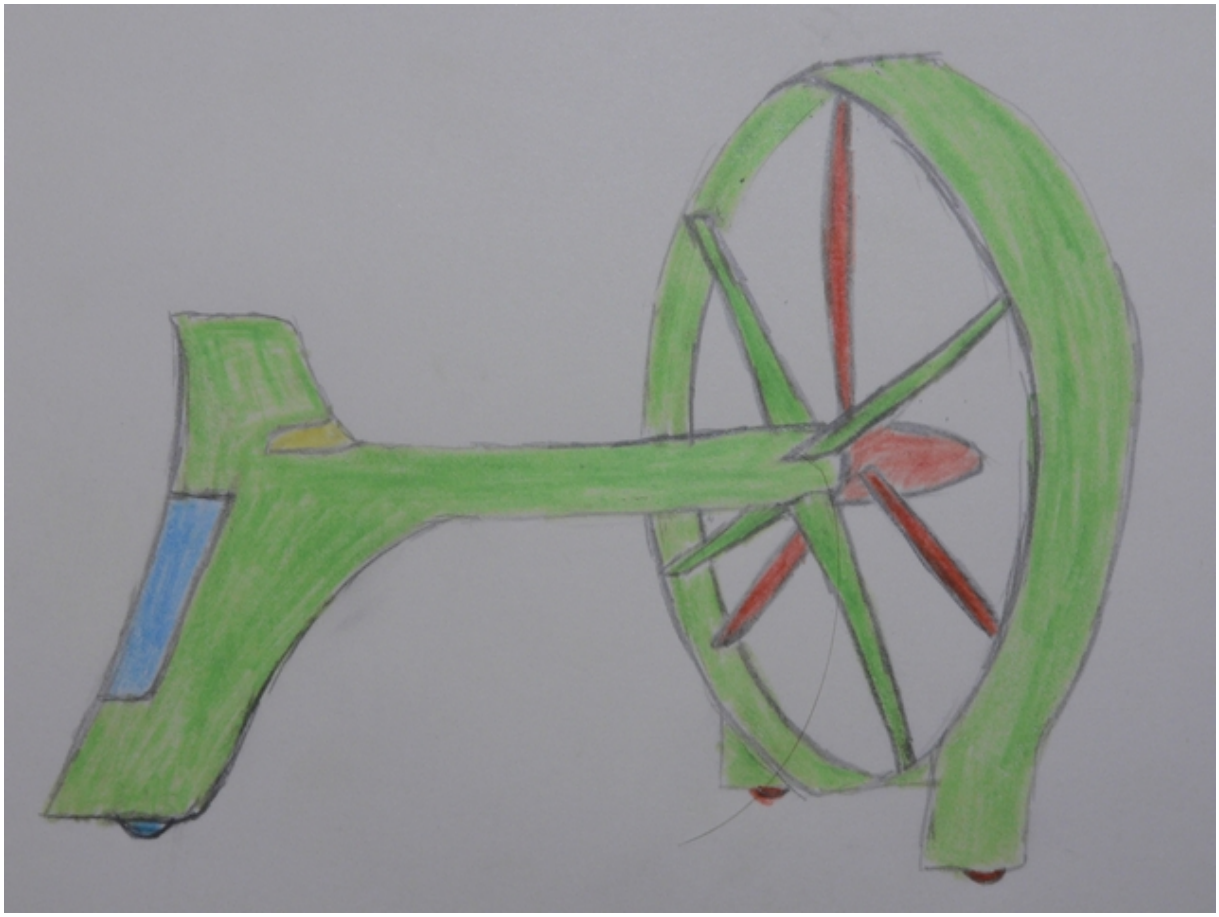
Acceleration of a record downwind craft propelling

Here it is shown that principally You can accelerate without using windmilling. At point 1 , that is standstill, the angle of attac is chosen so that the propeller is in full stall, providing nearly no lift but loads of drag. The aerodynamic force will pint to leeward but have a component perpendicular to the course and a component parallel to the course, driving the propeller forward like a spinnaker sail.
At points 3 to 6 the drag would not be able to drive the propeller anymore but acceleration can go ahead like before. The difference is that drag is not enough once You are fast enough so You need lift and high performance to continue acceleration to point 6 where steady stete is reached.

2.3 Prop missile

What is a prop missile?

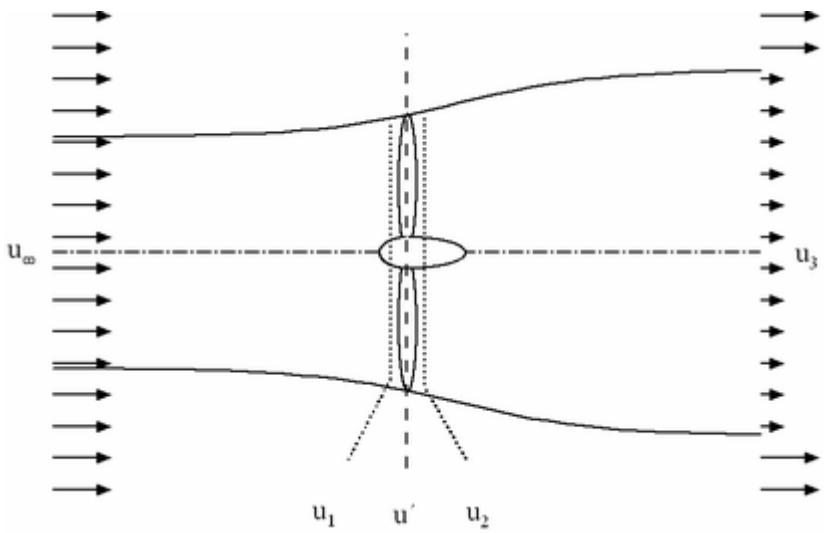
Well, it´s a means to launch a hot air balloon, go downwind and catch that balloon again for those who can´t kite.



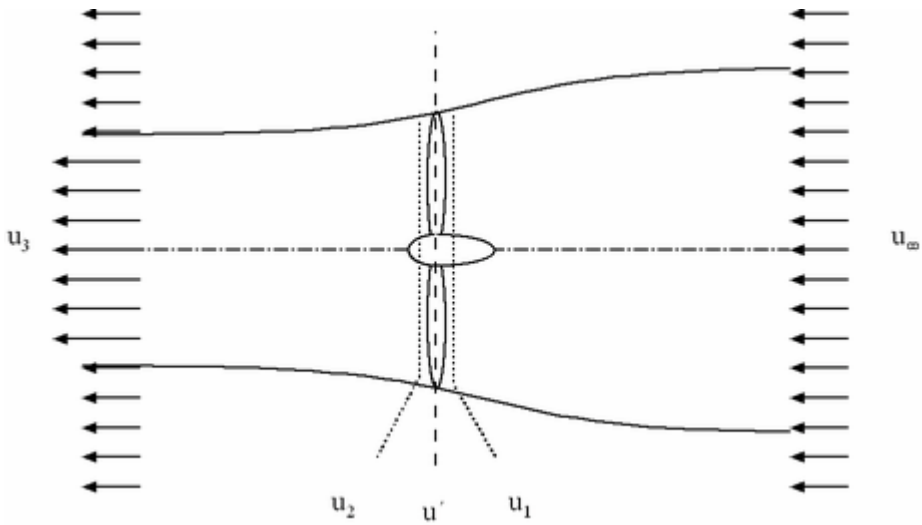
Prop missile

This rotor-stator concept should be "missile" enough.
The drive train drawn red would consist of the front wheels, transmission of the power to the ducted propeller that produces the thrust. Cockpit is Yellow and some blue rudder and rear wheel for control would be a good idea so You can find a pilot to enter the prop missile at free will.

Propeller theory will not be covered completely here, so please refer to [1],[2],[3],[4] or other literature on propeller theory.
Here, all i´d like to do is give a scetch of how propeller theory looks like. For producing thrust You have to accelerate air and so You will have to produce some slipstream. The velocity in the plane or the propeller is not identical to the undisturbed velocity but its the mean value $(U_\infty + U_3)/2$.
Propeller theory assumes, that the slipstream is close to constant all over the swept area of the propeller or repeller.
For we want to catch a hot air balloon we don´t care about specific values but can think big:
We take a large propeller in a very high sophisticated craft and build it so that we need only a minimum amount of slipstream. That means nothing but that we are free to choose a design specific load that doesn´t have to be high like in wind energy conversion or aircrat design. Efficiency is all that matters here.
Let´s see how it looks like:



Slipstream windmilling



Slipstream propelling

That said we can concentrate on thrust and power.
The power in the wheels of the prop missile is: *speed * thrust*
The power in the (marginal specific load) propeller is: *(speed - windspeed) * thrust*
The thrust is the same here for going downwind faster than the wind we can accelerate as slow as necessary as long as we accelerate at all.
The drivetrain betwen wheels and propeller and the propeller will have an efficiency η and that is taken into account here.
Let´s see what we get once we are faster than the wind and propelling:
Power in the wheels: speed * trust
Aerodynamic power: (speed - wind speed) * thrust
Since the wheels will have to drive the drive train and propeller we can calculate:
Aerodynamic power = η * *wheel power*
or: η * *speed* = *(speed - wind speed)*
and that leads to: ***speed = wind speed / (1 -*** η***)***
Let the efieciency of the vehicle be e.g. 80% (0.8) that would lead to a maximum possilble speed (or downwind VMG) of *5 * wind speed*.

All there is to check is wheather it´s possible to get there!
The start is easy: Look at a windmill and use the apparent wind to drive the wheels so thrust and wheel power (in windmilling mode *wheel power* = η * *aerodynamic power*) contribute to acceleration

until the apparent wind coes close to zero so friction and drag equal the driving force.
Now we are close to our aim. For there is no use in windmilling or propelling without a throughput in the propeller we need the slipstream! Close enough to wind speed, lets say 80% of the wind speed you need to have a slipstream of more than 20% the wind speed to get to propelling mode i.e. wind entering the propeller from the side You drive towards. Is this feasible?
Let´s see (now i take the slipstream into account!):
wheel power = 80% *wind speed * thrust*
*aerodynamic power = some small slipstream * thrust*
aerodynamic power must be lower than η * *wheel power* so:
some small slipstream < η * 80% *wind speed*
And that´s not at all impossible to design! Every wind energy converter in the world is deesigned like that so it should be possible to change dynamically from windmilling to propelling at a speed lower than wind speed.

Common sense and experience tell us the same thing:
Airplanes start their engines and produce the slipstream that is needed to produce trust.
Helicopters start their rotors and produce a thrust equalling their weight without any speed. The trroughput needed for the rotor is nothing but the slipstream itself before takeoff.
Last ut not least it´s possible to use a hair dryer even if You should use it in the wind (no matter heather the hairdryer is directed into the wind or leeward).

[1] Betz, Albert: Wind-Energie und ihre Ausnutzung durch Windmühlen, 1926
[2] Hütter, Wolfgang: Beitrag zur Schaffung von Gestaltungsgrundlagen für Windkraftwerke, 1942
[3] Glauert, H.: Windmills and fans. Aerodynamic theory, vol.4, Berlin 1935
[4] Wiederhöft, Klaus: 60 Jahre nach Hütter: Aerodynamische Radialschnitttheorie für Windenergieanlagen, DEWEK 2002

3 Math Appendix

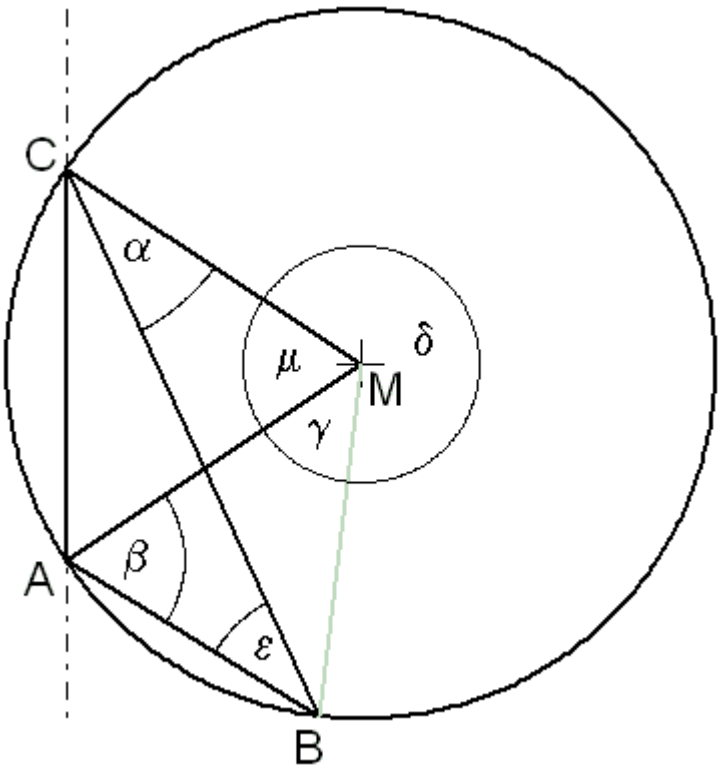
In Kitesurfing theory the *steady state performance circle* is a way to show most all steady state performance options of simple wind driven vehicles - including kiteboards - given the glide angle ε below 90° (high performance) and ε above 90° (low performance). The only case not covered by the performance circle is going dead downwind, that is: γ =180°.
Here´s the mathematical background.

Basic mathematical proofs
How to draw the performance circle
Points of special interest

3.1 Basic mathematical proofs

How to show that $\varepsilon = 1/2\ \mu$

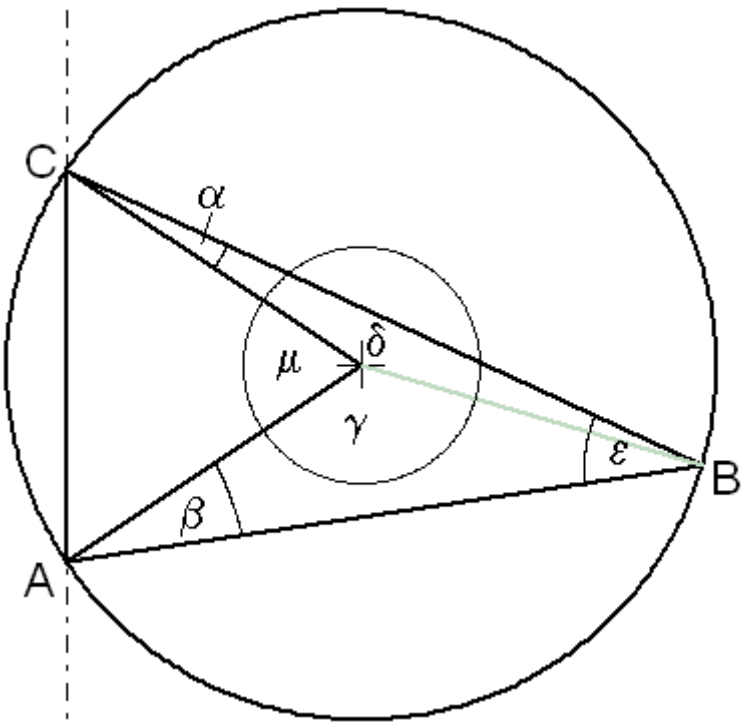
Case 1: Triangle *AMC* not inside of *ABC*



$\varepsilon < \pi/2$, High performance, Case 1

γ, δ and μ are angles in the center *M* of the circle. Thus the triangles *AMC*, *ABM* and *CMB* are isosceles triangles.
We find $\gamma = 180° - 2\ \beta$ and $\delta = 180° + 2\ \alpha$.
$\mu = 360° - \gamma - \delta$
Insert so we have: $\mu = 2\ \beta - 2\ \alpha$.
With the angle at *B* in *ABM* beeing β we can see that $\varepsilon = \beta - \alpha$.
Thus:
$\mu = 2\ \varepsilon$ or: $\varepsilon = 1/2\ \mu$.

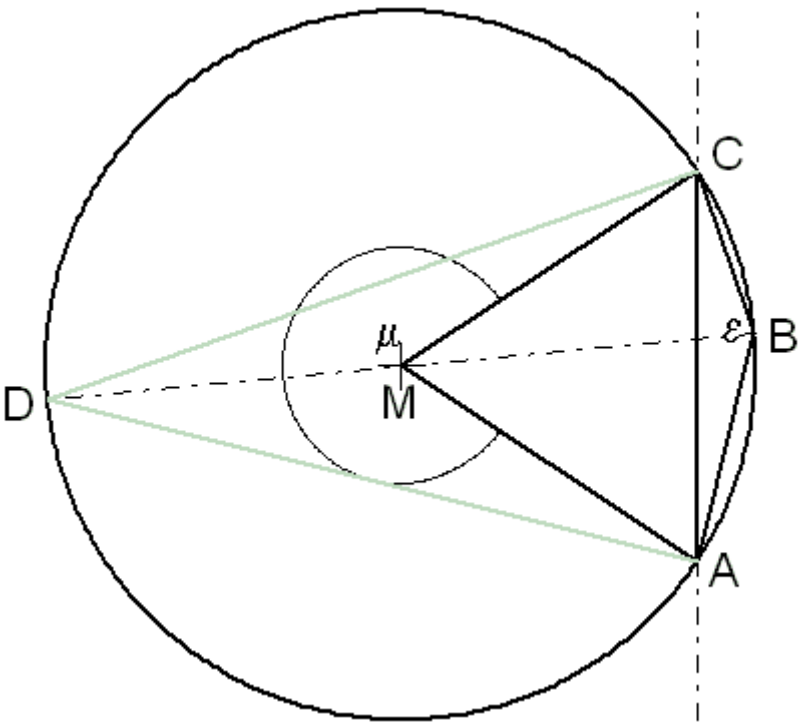
Case 2: Triangle *AMC* inside of *ABC*



$\varepsilon < \pi/2$, High performance, Case 2

γ, δ and μ are angles in the center *M* of the circle. Thus the triangles *AMC*, *ABM* and *CMB* are isosceles triangles.
We find the angles α and β at *B* where they add to ε: $\alpha + \beta = \varepsilon$
$2\alpha + \delta = 180°$ and $2\beta + \gamma = 180°$
that leads to: $2\alpha + 2\beta + \delta + \gamma = 360°$
insert $\alpha + \beta = \varepsilon$
$2\varepsilon + \delta + \gamma = 360°$
As $\mu + \delta + \gamma = 360°$ as well (center of the circle):
$\varepsilon = 1/2\,\mu$

Case 3: ε and μ not on the same side of *AC*



$\varepsilon > \pi/2$, low performance, Case 3

There´s only one case for $\varepsilon > \pi/2$.

The short solution for those who "see it":

$\varepsilon = 180° - \angle CDA$ or $\angle CDA = 180° - \varepsilon$ (See Thales' theorem in *DBC* and *DAB*).
$\mu = 360° - 2 \angle CDA$ (See analogy to case 2 above).
Insert and get: $\mu = 2\,\varepsilon$ or $\varepsilon = 1/2\,\mu$.

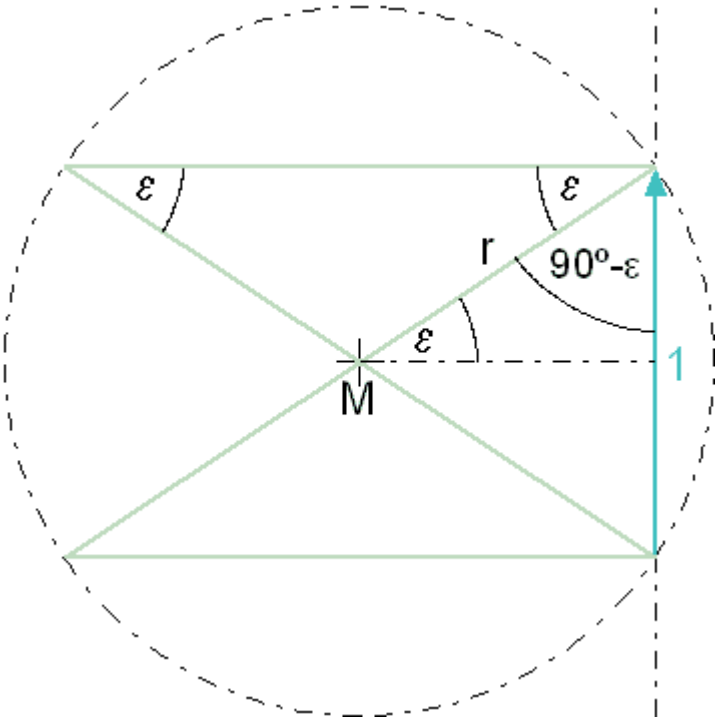
The complete step by step proof:

We draw a straight line through *B* and *M* so that *DB* is a diameter of the circle.
So we can use Thales' theorem: $\angle BCD$ and $\angle DAB$ are both 90°.
Furthermore we have an analogy to Case 2 that has already been proofed so we can use it: $\angle CDA = 1/2\ \angle CMA$.
The following triangles are isosceles: *DMC*, *AMD* and *CMA*.
Hence: $\angle MCD = \angle CDM$ and $\angle MDA = \angle DAM$.

To get ε we have to calculate the sum of the two other angles in ABC for $\varepsilon = 180° - \angle BCA - \angle CAB$.
Use Thales´ theorem in both *BCD* and *DAB*:
$\angle BCA + \angle CAB = 180° - \angle ACD - \angle DAC$.
This is the sum of the two right angles in *A* and *C* minus $\angle ACD$ and $\angle DAC$.
Insert this and we see:
$\varepsilon = \angle ACD + \angle DAC$.
In *ACD* there is: $\angle ACD + \angle DAC = 180° - \angle CDA$, so:
$\varepsilon = 180° - \angle CDA$.
Now in case 2 we saw that: $\angle CDA = 1/2\ \angle CMA$.
For $\mu = 360° - \angle CMA$ or $\angle CMA = 360° - \mu$ we can insert this to get: $\angle CDA = 180° - 1/2\,\mu$.
Hence: $\varepsilon = 1/2\,\mu$.

3.2 How to draw the performance circle

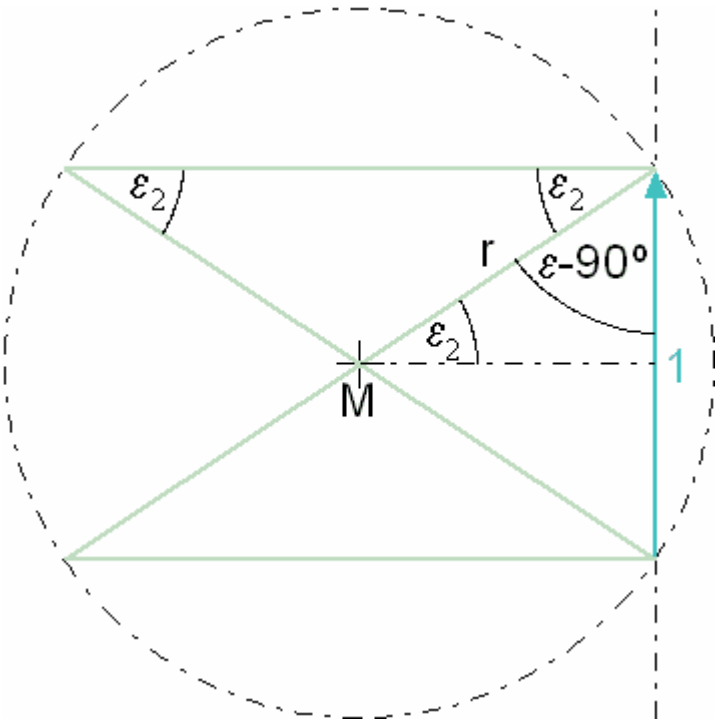
Use a set square and ε for ε < 90°



Easy enough to do

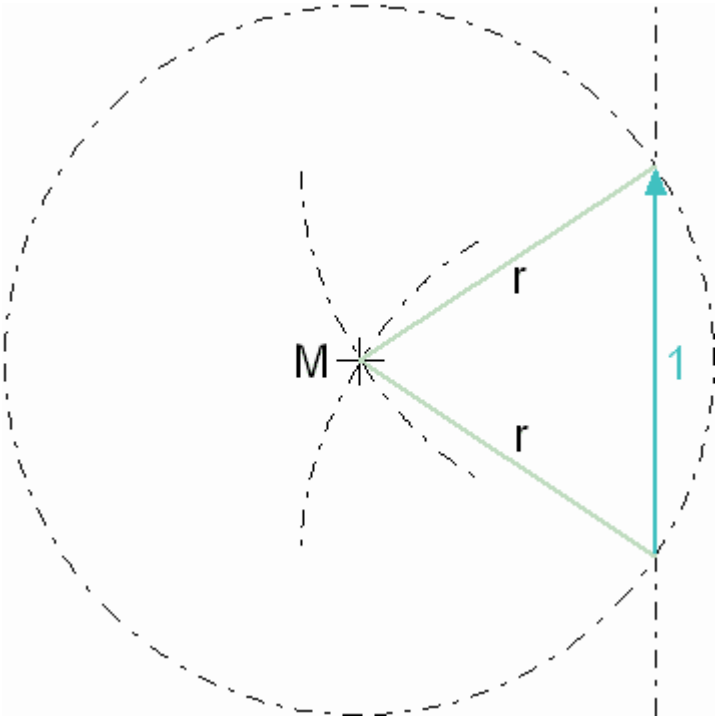
all You need to know or calculate is the angle at the tip and tail of the Wind vector.
Draw straight lines with an angle of 90°-ε at both ends and they will cross in the center M of the circle.

Use a set square and ε when ε > 90°



Use the corresponding glide angle

Use a pair of compasses and a calculator

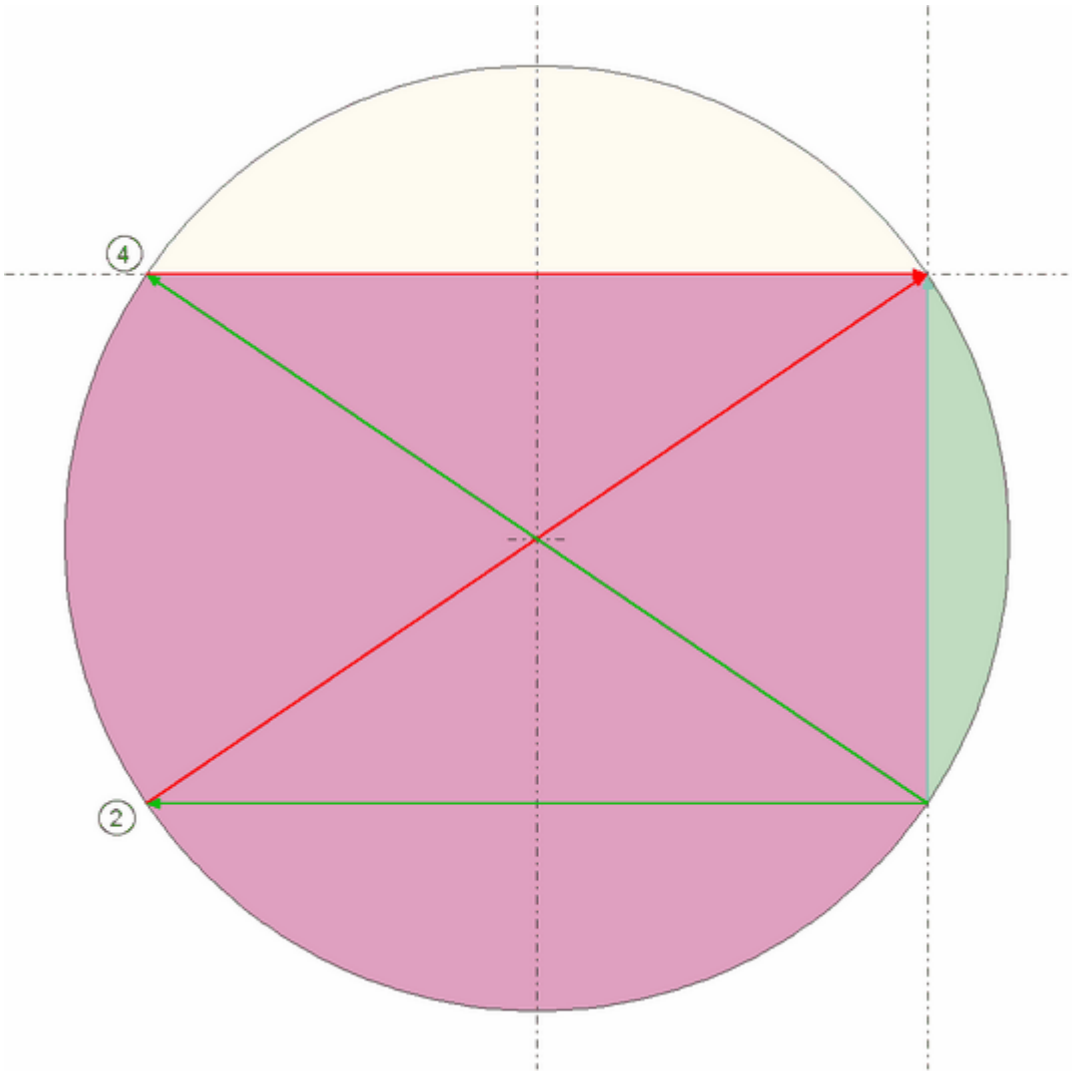


Nothing more to it than this

All You need is the radius of the performance circle. For the diameter is 1/sin(ε) the radius is r = 1/(2 sin(ε)).
There is no need to distinguish between high and low performance here because sin(180°-ε) = sin(ε).

3.3 Points of special interest

3.3.1 Maximum speed, -apparent wind



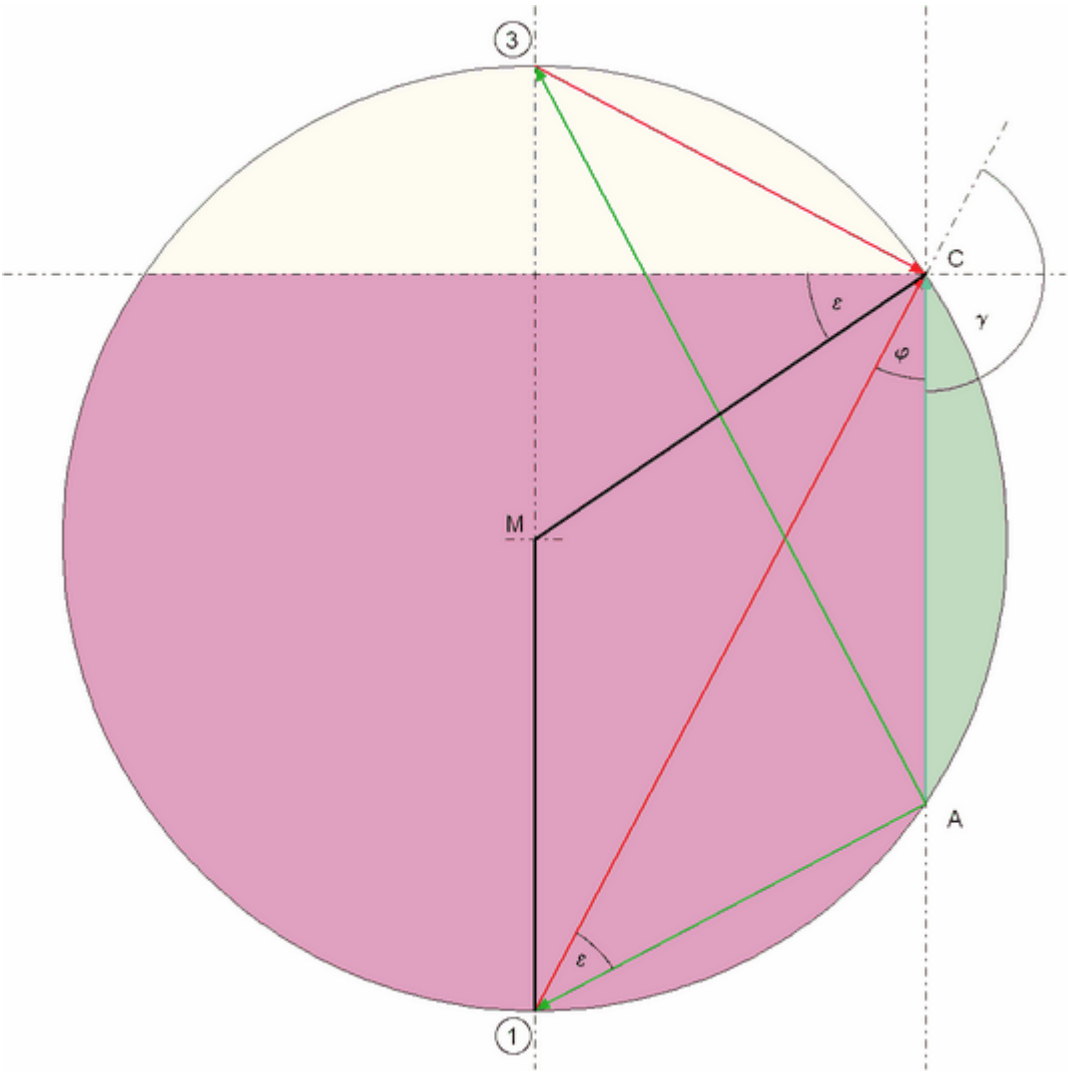
Diameters of the performance circle

Thales´ theorem makes these fairly simple.

4) Maximum speed:
Maximum speed is 1/sin(ε) times wind speed, the apparent wind is wind speed / tan(ε) and the course must be 90°+ε.

2) Maximum apparent wind:
Symmetry is obvious so speed = wind speed / tan(ε), apparent wind speed = wind speed / sin(ε) and the course is 90° (crosswind).

3.3.2 Maximum down-, upwind VMG



The centerline parallel to the wind

To understand maximum VMG i need to introduce a new angle ∠ *AC1*, that is φ = 180°-γ.
Remember, we want do catch the hot air balloon? So i start with maximum VMG downwind:

1) Maximum VMG downwind
For the center line going through *1*, *M* and *3* is parallel to *AC* ∠ *M1C* and ∠ 1*CM* and ∠ *AC1* are the same size.
So ∠ *ACM* = 2φ. We are half way there.
From the discussion of maximum speed we recall that the angle of the line between *C* and *M* has something to do with ε and so we write down: ε+ 2φ = 90° or φ = 45° - ε/2.
Now = γ = 180°-φ so insert φ and we get:
γ = 135°+ε/2.

Using the radius of the performance circle r = 1/(2 sin(ε)) it´s easy to write down the maximum VMG donwind:
Maximum VMG downwind = r + r sin(ε). Insert the radius and get:
VMG_max / wind speed = (1/sin(ε) + 1) / 2.

For the speed ratio and the apparent wind ratio we shall need the law of sines.

speed / windspeed = sin(∠ *1AC*) / sin(ε).
Use: ∠ *1AC* = 180°-ε-φ and insert:
speed / windspeed = sin(135°-ε/2) / sin(ε).

Apparent wind / wind = sin(45°-ε/2) / sin(ε) respectively.

2) Maximum VMG upwind
Thales´ theorem in *1C3* says that with maximum VMG upwind Your course is exactly 90° more upwind than with maximum downwind VMG:
γ = 45°+ε/2.

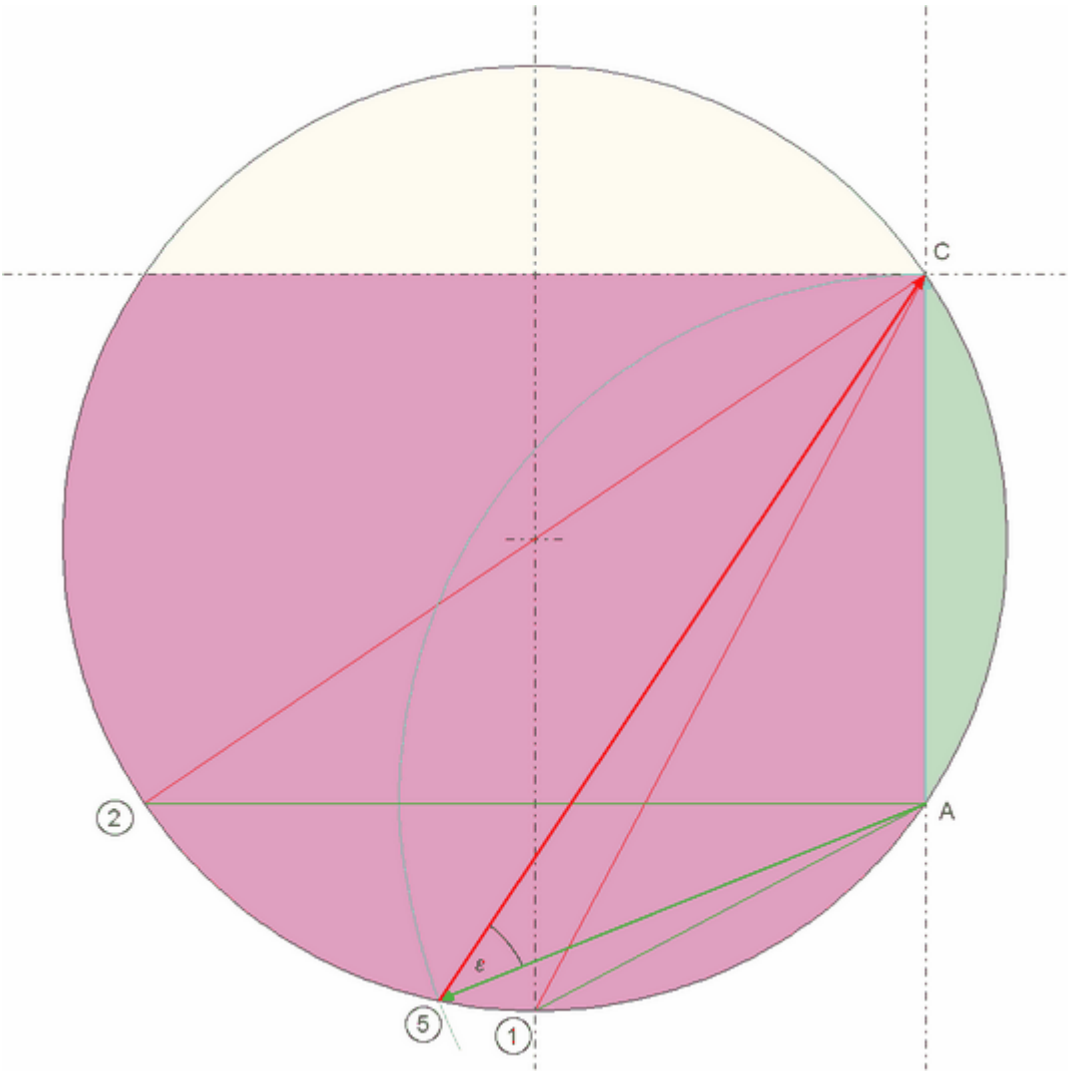
Due to the symmetrie of the performance circle the maximum VMG upwind must be exactly 1 times the wind speed lower than maximum VMG downwind:
VMG_max / wind speed = (1/sin(ε) - 1) / 2.

Like with VMG downwind the speed ratio and apparent wind speed ratio are calculated with the law of sines or simple taken from the downwind case using the symmetry:

Speed / windspeed = sin(45°-ε/2) / sin(ε).

Apparent wind / wind = sin(135°-ε/2) / sin(ε).

3.3.3 Apparent windspeed = windspeed



Looks difficult but gives an isoscale triangle

AC5 is an isoscale triangle. Thus the apparent wind speed and wind speed are the same and ε = 180°-γ.
For apparent wind speed is responsible for the kite power it might be quite interesting where on the performance circle Point 5 may be found.
5AC = *1AC* when *1AC* is isoscale so 180°-γ for optimum VMG downwind and ε must be the same: 180°- (135° + ε/2) = ε is true for ε = 30°. This may just be within reach for a good performing kite and board.
5AC = *2AC* when *2AC* is isoscale. This is the case when ε = 45°.
Every ε < 90° allowes for going at an apparent wind ratio of 1. You will find this ratio only with a downwind component in boardspeed. There´s no way to go upwind with apparent wind = wind speed as any upwind VMG would contribute to the apparent wind as well as the crosswind component of Your boardspeed will. Point Nr 5 will always be at the margin of the red sector of the performance circle. The smaller ε the higher γ will have to be.

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